GARRY ON PERSPECTIVE,

WITH

SIXTEEN

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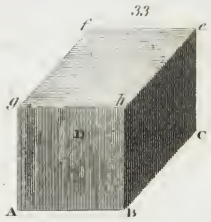
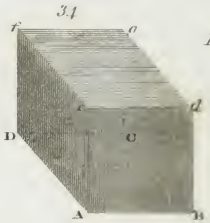
Francis Chester. 1830

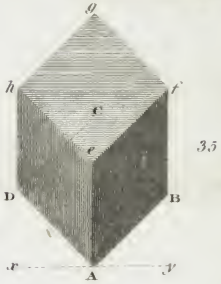
A

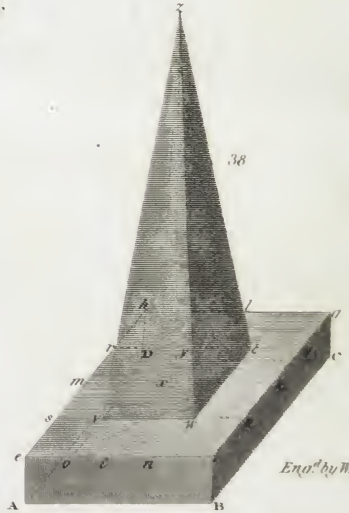
TREATISE ON PERSPECTIVE.

Plate 6









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A

TREATISE ON PERSPECTIVE.

DESIGNED

FOR THE USE OF SCHOOLS.

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ILLUSTRATED BY

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PREFACE.

The Science of Perspective has frequently engaged the attention of some of the most eminent mathematicians of Europe.

In projections of the sphere, the applications of this science are beautifully exemplified; whether in geographical or in astronomical investigations.

In representing the works of nature, or of art, perspective possesses most important advantages. In vain will the most glowing tints, conducted with the nicest judgment, endeavour to exhibit the captivating beauties of nature, without the magic aid of perspective.

I assure the reader I have experienced many anxieties of mind in obtaining whatever knowledge I possess of this curious

branch of study. I have pored with patient attention over the labours of those who are now no more; but whose works will live for ever.

I acknowledge with gratitude the advantage I have derived from the judicious and scientific remarks of Mr. Peter Nicholson, author of the Architectural Dictionary, and of many other popular works.

I have invariably observed, that the language of oral instruction, and the language of books, in general, produces very different effects.

In the former, there is occasion for a seeming tautology, or constant repetition, which is an extreme usually avoided in the latter.

Having never seen any treatise arranged according to my present ideas on the subject, I have attempted this little performance, wholly from memory, without consulting any author: wherefore, I beg the ingenious reader will, if errors occur,

favour me with communications. I know not what may be its fate, but I confess I have done the best in my power to render the subject intelligible; and therefore rely on that British candour and liberality which I have experienced for years; and which, during life, can never be erased from my heart.

Lest the perspective arrangement of this little work be thought anomalous, I beg to state, that this peculiar order is an attempt to facilitate the acquisition of the science.

Through some of the questions, &c. for practice, it is proposed or required to construct a figure twice or double the size of a given figure: in such cases I simply mean the required figure to be constructed on a line twice the length of some line in the given figure; for I do not suppose *all* our readers to be acquainted with the duplicate or triplicate ratios of magnitudes.

In order to view a perspective drawing,

it is requisite to keep the eye over or opposite the centre of the picture; and distant from it, as far as the eye is placed in the drawing from the picture: then bringing the hand to the eye, with the fingers closed, so as to leave sufficient opening to view the whole drawing, the representation will then appear true to the original. But when the distance is less or shorter than seven or eight inches, this method is almost impracticable.

4, Southampton Crescent, Euston Square, Somerstown.

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TREATISE ON PERSPECTIVE.

PRACTICAL GEOMETRY.

GEOMETRY, from the Greek compound word *gai*, or *ge*, the earth, and *metro*, to measure, signifies to measure the earth, or to measure land.

Egypt has been formerly the seat of universal learning; and thither the Greeks often resorted for literary instruction.

The Nile, a river about fifteen hundred miles long, annually overflows the land of Egypt: beginning on the 15th of June, and continues for about three months before it begins to decrease. On the overflowing of the Nile depend the fertility and the crops of Egypt. In the early ages of the world, the Egyptians used certain marks to distinguish the limits or boundaries of each family's portion of land; but, by the inundation of the Nile, those landmarks were usually carried away: hence the

natives were obliged to have recourse to actual mensuration: and this practice first gave rise to the word Geometry. But geometry, at present, is more general in its signification, as it relates to points, lines, surfaces, solids, &c.

A point. A point has position but no magnitude; that is, strictly speaking, a point is supposed so small as to become invisible: and yet it is said to have position.

PLATE I.

A point is usually noted by a letter, as the point A, No. 1.

A line. A line is length, without either breadth or depth. That is, a line must have length, but neither breadth nor depth.

A line is usually noted by two letters, one at each end, as the line A B, No. 2.

An angle. Angle is the inclination of two lines to each other.

An angle is usually noted by three letters; the second, or middle letter, is at the angular point: thus, we say the angle A B C, or C B A, No. 4.

Sometimes a single letter denotes an angle; as we say the angle B.

Radius. Radius is a Latin word, and means a sunbeam; because it issues or comes from

the sun, as from a centre: and hence, we open a pair of compasses to an extent, and placing one foot in any particular point, we say, with such a radius, and from such a centre, describe an arc, or an arch. Thus we say (No. 3), with the radius A *c*, and centre A, describe the arc *c d*.

A perpendicular. This is a compounded Latin word from *per*, by, and *pendo*, or *pendeo*, to weigh or suspend; for a weight made fast to one end of a string, while the other end is supported, will make the line or string perpendicular to the horizon, that is, to the floor, or ground.

To raise a Perpendicular from a given Point, in a given Line.

Case I. When the given point is near the middle of the line.

Let A B, *fig.* 5, be the given line, and D the given point: open the compasses to any convenient extent, as from D to *e*, and make D*f* equal to D*e*; then with one foot of the compasses in *e*, open the other foot to any extent more than half *ef*, and describe an arc at C; with the same radius, and centre *f*, cross the former arc in the point C; through D and C, draw the line D C; which is the perpendicular required.

Because the angle A D C is equal to the adjacent angle B D C; the line D C is perpendicular to A B.

CASE II. When the given point is near one end of the given line.

Let A B, *fig.* 6, be the given line, and C the given point: place one foot of the compasses in the point C, open the other to any convenient extent, as to *e*, with the radius C *e*, and centre C, describe the arc *e f g:* now, without opening or closing the compasses, lay off the radius C *e* from *e* to *f*, and from *f* to *g;* with the same radius and centre *f*, describe an arc at D; then with the same radius and centre *g*, cross the former arc in the point D, and draw D C the perpendicular required.

To let fall a Perpendicular; that is, from any given Point above or below a given Line to draw a Line perpendicular to such given Line.

Let A B, *fig.* 7, be the given line, and C the given point: place one foot of the compasses in the point C, and open the other foot below the line A B; then, from the centre C, describe the arc *f y e;* with the same radius, or any extent more than half *e f*, and centre *e*, describe an arc at *x;* with the same radius and centre *f*, describe an arc, crossing the former in the point

x; draw C *x*, cutting A B, in the point D; and C D is the perpendicular required.

CASE II. When the given point is nearly over one end of the given line.

Let C, *fig.* 8, be the given point, and A B the given line. A little to the right of the given point, as in the present case (but to the left if the greater part of the line had been on the left), take any two points as *e*, *f*; then, with the radius *f* C, and centre *f*, describe the arc C *m x*; again, with the radius *e* C, and centre *e*, describe an arc crossing the former arc in the point *x*; draw C *x*, cutting A B in the point D; and C D is the perpendicular required.

Bisect is a compounded Latin word, from *bis*, twice, or two, and *seco*, to cut, that is, to divide into two equal parts.

To bisect a given Line.

Let A B, *fig.* 9, be the given line, it is required to divide it into two equal parts, that is to find the middle of A B.

Place one foot of the compasses in the point B, and with any radius more than half B A, describe arcs at *x* and *y*, with the same radius and centre A, describe arcs crossing the former arcs, in the points *x* and *y*; draw *x y*, cutting A B in C, the point of bisection, or middle of A B, as required.

Angle, from the Latin angulus, means a corner, &c.

To make an Angle equal to a given Angle.

Let A B C, *fig.* 10, be the given angle, it is required to draw two lines, making an angle equal A B C.

Draw any line as B C, *fig.* 11; with any radius, as B *d*, *fig.* 10, describe the arc *d e*; then with the same radius and centre B, *fig.* 11, describe the arc *d f*; take in the compasses the arc *d e*, *fig.* 10, and lay off that extent from *d* to *e*, in *fig.* 11; through *e* draw B *e* A, and A B C is the angle required.

Because the radius B *d*, in *fig.* 10, is equal to the radius B *d*, in *fig.* 11, and the arc *d e*, in *fig.* 10, equal to the arc *d e* in *fig.* 11, we say the angle A B C, *fig.* 10, must be equal to the angle A B C, *fig.* 11.

To bisect a given Angle, that is, to divide it into two equal Angles.

Let A B C, *fig.* 12, be the given angle.

With any convenient radius as B *e*, and centre B, describe the arc *e g*; with the same radius and centre *e*, describe an arc at D, with the same radius and centre *g*, describe an arc, crossing the former in the point D, draw B D, cutting the arc *e g* in the point *f*, and the

angle A B D is equal to the angle D B C as required.

To find a Centre for three given Points, but the given Points cannot be in a straight Line.

Let A, B, C, *fig.* 13, be the three given points, it is required to find the centre of a circle, whose circumference will pass through the given points A, B, C.

Join A B, and join B C, bisect A B and B C, as in *fig.* 9, and through the points *d* and *e*, where the arcs cross each other for A B, draw *d e;* then through the points *f* and *g*, where the arcs cross each other for B C, draw *f g*, which produced or drawn further will cross *d e*, in *o;* and *o* is the centre required: for, place one foot of the compasses in *o*, and open the other to any one of the given points, and the arc of a circle may pass through them, as required.

To divide a given Line into any Number of equal Parts.

Let A B, *fig.* 14, be the given line, it is required to divide it into five equal parts.

Draw the line A C, making any angle with A B; draw another line from B, as B D making the angle A B D, equal to the angle B A C, as in *figs.* 10 and 11: then take any convenient

extent in the compasses, and lay it off five times from A toward C, and from B toward D, then join the respective divisions in A C to the divisions in B D, and the lines so drawn will divide A B as required: which may be seen in the figure.

Triangle, from the Latin, means a figure of three sides and three angles.

Equilateral triangle, from the Latin *æquus*, equal, and *latus*, a side, that is, a triangle having equal sides, and when so it must have equal angles.

Second Method of dividing a Line into any Number of equal Parts.

Let A B, *fig.* 15, be the given line, it is required to divide it into six equal parts.

Draw any line as D E, *fig.* 16; take any convenient extent in the compasses, and lay it off six times from D toward E: here the sixth division ends at 6: with the radius 6 D, and centre 6, describe the arc D C; with the same radius and centre D, describe an arc crossing the former in the point C; through the points C and D, draw C D, indefinitely below D E: draw also C 6: here the triangle D C 6 is an equilateral triangle, because the three sides are equal, and the three angles are also equal.

Take in the compasses the given line A B,

fig. 15, and lay it from C to A, in *fig.* 16, and also from C to B. Here the given line is laid on each side of the triangle produced; now if we join AB, this line AB will also be equal to the given line AB, *fig.* 15; because ACB, *fig.* 16, is equilateral; hence, if we produce lines drawn from C, through D6, as numbered in the figure, they will also divide AB into six equal parts as required.

Let the pupil divide a line, twice as long as AB (*fig.* 14), into seven equal parts.

Required, to divide a line twice as long as AB, *fig.* 15, into nine equal parts, each by the two foregoing methods.

Diameter, from a Greek word *dis*, twice, or double, and *metron*, measure, that is, double the radius, or double the area, means a line drawn through the centre of a circle, meeting the curve or circle in opposite points, which points are said to be diametrically opposite.

To find the Centre of a Circle.

Plate. 2,

Let CABD, *fig.* 17, be a given circle, to find its centre.

Draw any line, as AB, bisect AB, as in *fig.* 9, plate 1, and draw *nm* indefinitely, and *rs* is a diameter: again, bisect *rs*, and draw CD, crossing *rs*, in the point *o*, which is the centre of the circle as required.

A square is a figure which has four equal sides, and four equal angles. All right angles, or 90^0 each.

One Side being given to construct the Square.

Let AB, close to *fig.* 18, be the given side of the square.

Draw any line as A*e*, *fig.* 18, draw AD perpendicular to A*e*, as in *fig.* 6; lay off the given line AB, from A to B, and from A to D; with the radius AB, and centre B, describe an arc at C; with the same radius and centre D, cross the former arc in C, draw CD and CB, and ABCD is the square required.

Parallelogram, from the Greek *para*, at, *allelon*, mutually by turns, and *gramma*, a figure, or a line, all mean a figure, whose sides are mutually parallel, so that all perpendiculars from one side to its opposite side will be equal.

To construct a Parallelogram, two of its Sides being given.

Let *m* and *n*, close to *fig.* 19, be the two given sides.

Draw any line as A*e*, *fig.* 19, raise AD perpendicular to A*e*, as in *fig.* 6; lay off the line *n*, from A to B, and the line *m*, from A to D; with the radius AB, and centre D, describe an

arc at C; with the radius AD, and centre B, describe an arc crossing the former arc in C, draw CD, CB, and ABCD is the parallelogram required.

To make a Triangle, the Three Sides being given.

Let *l*, *m*, *n* be the three given sides, a little above *fig.* 20.

Draw any line as AB, *fig.* 20; lay off the line *n* from A to B; with the line *m*, in the compasses, and one foot in A, describe an arc at C; with the line *l*, in the compasses, and one foot in B, describe an arc crossing the former in C, and these joined, ABC is the triangle required.

Note. When three lines are given to construct a triangle, the length of any two of such lines must be greater or longer than the length of the third line; otherwise, the triangle cannot be constructed.

OF THE CIRCLE.

Circle, from the Greek word *kuclos*, round, means a figure of circular form like a hoop.

Circumference, from the Latin *circum*, about, and *fero*, to bear or carry; that is, to move about, for a circle is made by placing one foot

of the compasses in a point, as a centre, and moving the other foot round to form the circle.

DIVISIONS OF THE CIRCLE, ETC.

In the early ages of the world, the shepherds and others of Egypt, and of the contiguous nations, from the serenity of their sky, often contemplated, in their nightly watches, the various appearances of the heavens. From attentive observations, they concluded the earth revolved round the sun in 360 days; and by this circumstance, it is supposed, the circumference of every circle was divided into 360 equal parts, called degrees. The Latin for degree is *gradus*, and gradus means a step or movement, alluding to the movement of the earth in its orbit round the sun.

Now, as circles may be of various magnitudes, it was thought proper to divide a degree into 60 equal parts, called minutes, and a minute into 60 equal parts, called seconds, and so on to thirds, fourths, and fifths; but for ordinary purposes, the division of a degree into 60 minutes, is thought sufficiently correct.

The names, degrees and minutes, are abbreviated thus, $15^0\ 15'$; here beginning at the left, we say, 15 degrees, the little ^{0} on the right of

15 standing for degrees; the next in order is 15 minutes, noted by the mark ′ on the right of the second 15.

If we draw a diameter as AB, *fig.* 21, and another diameter at right angles to AB, as *ef*, these two diameters divide the circle into four equal parts; and consequently divide 360° into four equal parts; hence, each fourth part contains 90°; because four nineties make 360°.

Any two lines drawn from the centre of a circle to its circumference will include a certain number of degrees, minutes, &c.

The two lines *o*A and *ox* contain, in the arc A*x*, 28° 30′: and the two lines *o*A and *oe* contain, in the arc A*e*, 90°.

The line *eo* is perpendicular to A*o*, and the angle A*oe* is called a right angle, because it contains 90°.

Any angle less than a right angle, is called an acute angle; as the angle A*ox*, which contains only 28° 30′.

Any angle greater than a right angle, is called an obtuse or open angle, as the angle B*ox*, which contains the right angle B*oe*, equal 90°, and the acute angle *eox* equal 60° 30′, and 90° added to 60° 30′ make 150° 30′, equal to the angle B*ox*.

What any angle wants of 90° is called its complement; thus, the complement of A*ox*,

equal 28^0 30′, is *xoe* equal 61^0 30′; because 61^0 30′ added to 28^0 30′ make 90^0.

What any angle wants of 180^0 is called its supplement; thus, the supplement of the angle BO*x*, equal 150^0 30′, is the angle A*ox*, equal 28^0 30; because 150^0 30′ added to 28^0 30′ make 180^0; the number of degrees in every semicircle.

QUESTIONS FOR PRACTICE.

What is an acute angle?

What is an obtuse angle?

What is the difference between an obtuse and an acute angle?

Is the angle 89^0 59′ acute or obtuse?

Is the angle 179^0 15′ acute or obtuse?

What is the complement of an angle?

What is the complement 89^0?

What is the complement 5^0 55′?

What is the supplement of an angle?

What is the difference between the complement and the supplement of an angle?

What is the supplement of 50^0 59′?

What is the supplement of 124^0 10′?

What is the supplement of 90^0?

OF SCALES.

We require but few words to explain the scales we shall have occasion to use.

There are various scales, divided as may suit the convenience of the artist, workman, &c. On the common two feet rule, an inch is divided into eight parts, and here if one inch represent a foot, $\frac{2}{8}$ will represent $\frac{1}{4}$ of a foot, or 3 inches; $\frac{4}{8}$ half a foot, or 6 inches; $\frac{6}{8}$ three-fourths of a foot, or 9 inches.

Again, if one inch be divided into 12 equal parts, every small division will represent one inch. In like manner, if $\frac{3}{4}$, $\frac{1}{2}$, or $\frac{1}{4}$ of an inch be divided into 12 equal parts, every large division will respectively represent one foot, and every small division will be an inch.

But the scales we shall have occasion to use are those in which the inch, half-inch, and quarter inch, are respectively divided into ten equal parts. In this case, every large division is called a foot, and the small divisions tenths of a foot. Besides the convenience of these divisions for decimal operations, we may also easily ascertain certain divisions corresponding nearly to any number of inches, thus for 5

feet we take five large divisions, whether 1 inch, $\frac{1}{2}$ inch, or $\frac{1}{4}$ inch.

Again, for 5 feet 3 inches we take five large divisions and two tenths, and one-half the next tenth.

For 5 feet 6 inches, we take 5 feet 5-tenths.

For 5 feet 9 inches, we take 5 feet 7-tenths, and one-half another tenth.

From these remarks, with a little attention, we may determine the divisions for 4, 5, 7, 8, 10, and 11 inches.

The above scales, by employing the diagonal divisions, may be employed for three numbers, or for hundreds; but the above will suffice for our present purpose.

THE LINE OF CHORDS

Is also laid down on the plane scale: it begins on the left, with the word chords, ch. or c. The word chord comes from the Greek *chorde*, and means the intestine of an animal of which musical strings used to be made: and as the chord of any arc or angle is always a straight line, it represents a musical string, in the act of tension, or being stretched out.

The line of chords consists of nine primary divisions, numbered 10, 20, &c. up to 90. The spaces between the primary divisions are again

divided into ten parts, making in all, from the beginning to 90, 90 divisions: and, on large scales, the degrees are again subdivided.

The radius of any circle is always equal to the chord of 60^0 of that circle; and whenever we have an angle to measure, or an angle to make, and to contain a certain proposed number of degrees, we must always take the chord of 60^0 in the compasses.

Required to measure the Angle A *o x, fig.* 21.

Here the radius *o* A is too short for the line of chords on the plane scale, wherefore I produce *o* A toward *h*, and *o x* toward *n*; then open the compasses, so that one foot may fall on the beginning of the line of chords, and the other foot on 60^0; then, with this radius and centre *o*, describe the arc *h n:* I take the arc *h n* in the compasses, and apply it from the beginning on the line of chords, and it determines the measure of the angle to be 28^0 30′.

Again, required to lay off the Angle of 28^0 30′.

Draw any line as *o h* (in *fig.* 21), then with the chord of 60, taken from the line of chords in the compasses, with one foot in *o*, describe the arc *h n*; take from the line of chords the chord of 28^0 30′, and lay off that extent from

h to *n*, draw *on*, and *hon* is the angle required, containing 28° 30′.

If the required angle contain more than 90°, take any two chords successively, the sum of which will be equal to the required number; and the angle may be laid off.

Suppose the required angle to contain 140° first from the angular point determined, describe an arc with the chord of 60°; then take the chord of 70°, and lay it on the arc twice; through this last point draw a line, as in the foregoing operations, and the angle is laid off as required.

PROPOSED EXAMPLES FOR PRACTICE.

Required to make an angle containing 30°.

Required to lay off an angle containing 58° 30′.

Required to lay off an angle containing 120°.

Here note that by drawing a straight line, as *h* B, *fig.* 21, and taking a point about the middle of it, and then describing a semicircle, we may easily lay off any angle more than 90°. Thus suppose it required to lay off an angle of 151° 30′.

Its supplement is 28° 30′, which I lay off from *h* to *n*, and the required angle B*ox* is constructed.

Required to lay off one angle of 175°, and another of 108° 30′, by this method.

OF POLYGONS.

Polygon, from the Greek *polus*, many, and *gonia*, an angle, means all figures having a greater number of sides than a square. Such as a figure of five, six, seven, or more sides, and having, respectively, as many angles as sides.

When the sides of a polygon are unequal, the angles are also unequal, and this we call an irregular polygon.

When the sides of any polygon are respectively equal, the angles are also equal, and this we call a regular polygon.

OF REGULAR POLYGONS.

If we include the triangle and square, the following are the names of the principal polygons:—

A trigon, from the Greek *tris*, thrice or three, and *gonia*, a corner or an angle, is a figure of three equal sides, and three equal angles.

A tetragon, from *tetros*, four, and *gonia*, an angle, is a figure of four equal sides, and as many equal angles.

A pentagon, from *penta*, five, and *gonia*, an

angle, is a figure of five equal sides, and five equal angles.

A hexagon, from *hex*, six, and *gonia*, an angle, is a figure of six equal sides, and as many equal angles.

A heptagon, from *hepta*, seven, and *gonia*, an angle, is seven equal sides, and as many equal angles.

An octagon, from *octo*, eight, and *gonia*, an angle, is a figure of eight equal sides, and as many equal angles.

A nonagon, from the contracted Latin word *nona*, nine, and *gonia*, an angle, is a figure of nine equal sides, and nine equal angles.

Decagon, from the Greek *deca*, ten, and *gonia*, an angle, is a figure of ten equal sides, and as many equal angles.

There are more regular polygons, but the above are sufficient for our present purpose.

To inscribe a Polygon within a given Circle.

Rule.—Divide 360^0 by the number of sides in the required polygon, and the quotient will give the angle at the centre; then draw a line from the centre to the circumference, take with the compasses, from the line of chords, the chord of the angle at the centre, and lay off this extent on the circle from the line last drawn,

through this point draw another line from the centre; the ends of these lines joined will be the side of the required polygon: which applied round the circle, will mark out the sides of the regular polygon.

Note. If the given circle be too small for the chord of 60°, taken from the scale, produce indefinitely one line from the centre; then with the chord of 60° describe an arc, lay off the chord of the angle at the centre on this arc; through this last point draw another line from the centre, which cuts the given circle in its proper point, join these two points, which is the side of the polygon required. See *fig.* 22 *plate* 2. And if the given circle be too large for the chord of 60° on the scale, draw one line from the centre to the given circle; then with the chord of 60° describe an arc, on which, from the line drawn, lay off the chord of the angle at the centre; through this point draw another line, produced to meet the given circle, these two points joined in the given circle, form the side of the polygon required.

EXAMPLE.—Let AEB, *fig.* 22, be a given circle, it is required to inscribe within it an equilateral triangle.

Here, 360° divided by 3, the number of sides, gives 120°, the angle at *o* the centre; but, the given circle is too small for the chord of 60° on

the scale; wherefore, I produce a line as *o* A indefinitely toward *m;* then, with the chord of 60°, describe an indefinite arc, and lay off on it the chord of 60° from *m* to *y*, and from *y* to *n*, which make 120°: through *n* draw *o* B *n*, and *mon* is the angle at the centre, equal 120: join A B, which is the side of the required triangle: for, this taken in the compasses, goes round the circle just three times, as in the figure: hence, A E B is the triangle required.

PROPOSED EXAMPLES FOR PRACTICE.

Required to inscribe a trigon, or equilateral triangle, within a circle, whose radius is four feet, from a scale of one half inch to a foot.

To inscribe a square in a circle whose diameter is six feet, from a scale of one half inch to a foot.

To inscribe a pentagon in a circle whose radius is three feet, from a scale of one quarter of an inch to a foot.

To inscribe a hexagon in a circle whose semidiameter is two feet, from a scale of one inch to a foot.

To inscribe a heptagon, an octagon, and a nonagon, each within a circle whose diameter is ten feet, from a scale of one quarter of an inch to a foot.

The foregoing being three distinct operations, require three distinct circles.

The Side of a Polygon being given to construct the Polygon.

Rule.—The first part of this operation is wholly performed by the mind.

Divide 360^0 by the number of sides in the required polygon, and the quotient is the angle at the centre; this angle taken from 180^0, leaves the angle of the polygon, that is, the angle made at the circumference of the circle by two sides of the polygon: but for the construction we want half the angle of the polygon, because we have to construct a triangle, the base of which is the given side of the polygon, and the angles, at the base, will each be half the angle of the required polygon.

Every triangle contains 180^0: now, if we have the angle at the centre for any regular polygon, we can easily find the sum of the two angles at the base; and half this sum will be the degrees contained in each angle at the base; which, being known, with the chord of 60^0 in the compasses, and one foot in one end of the given side, describe an arc; with the same radius, and one foot in the other end of the given side, describe an arc in like manner; then lay off on each arc the degrees, &c. in the angle at the

base, through each of these points draw lines from each end of the given side, and where they cross or intersect will be the centre of a circle within which the required polygon is to be drawn; with one foot in this centre open the other to either end of the given side, describe a circle, and the given side will go round the circumference as many times as there are sides in the required polygon.

Note. If the given side of the polygon be less or shorter than the chord of 60° on the scale, produce the given side both ways, first marking distinctly each end of the given side.

EXAMPLE.—Let AB, *Plate* 3, *fig.* 23, be a given side of a trigon, or equilateral triangle, it is required to construct the triangle by the above rule.

First, divide 360° by 3, and the quotient is 120°, this subtracted from 180°, leaves 60°; the half of 60° is 30°, the angle at the base. Here the given side is too short for the chord of 60°, so I produce it both ways, then with the chord of 60° and centre B, describe the arc *de*, with the same radius and centre A describe the arc *fg*; lay off the chord of 30°, from *d* to *e*, and from *f* to *g*, draw A*g* and B*e*, cutting A*g* in the point *o*; the centre of the circle required. With the radius *o*A, or *o*B, and centre *o*, describe the circle A*ih*B: here AB will go three

times round this circle, viz. from A to B, from B to C, and from C to A; draw the sides as in the figure, and A B C is the required figure or trigon.

EXAMPLES FOR PRACTICE.

Given the side of a trigon, three feet long, from a scale of one half inch to a foot, to construct the trigon.

The side of a square, four feet long, scale as in the preceding, to construct the square.

The side of a pentagon, three feet, from a half inch scale, to construct the pentagon or figure of five sides.

The side of a hexagon, three feet, from a quarter inch scale, to construct the hexagon.

The side of a heptagon, of an octagon, and nonagon, each three feet, from a quarter inch scale, to construct each polygon separately.

A Line being given, and a Point above or below it through which to draw a Line parallel to the given Line.

Let A B, *fig.* 24, be the given line, and C the given point, above A B.

Place one foot of the compasses in the point C, open the other foot so that in describing an arc, the extremity of the arc, as *y*, may coincide with A B; then, with the same radius, take any point in A B for a centre, as the point *o*, and de-

scribe an arc, as Px; through the given point C, and the upper part of the arc Px as i, draw CD, which is parallel to AB, as required.

Second Method.

Let AB, *fig.* 25, be the given line, and C the given point.

From C draw a line to AB, as Cy, making any angle with AB, as CyB; then, from any point in AB, as x, draw xD, making the angle DxB, see *figs.* 10 and 11, equal the angle CyB, make xD equal yC, draw CD, which is parallel to AB as required: or make DCy equal CyA.

This may be readily done with a parallel ruler, by placing the edge of the upper half to AB, then press the lower half; move the upper edge to C, now press the upper half, and draw CD, which is the parallel to AB as required.

OF PLANES.

A plane is a level extended surface, as a floor, or a flat ceiling, and having no depth or thickness.

Every plane has a surface, but every surface is not a plane, for a hill or a globe has a surface, but that surface is not a plane, because such a surface is not level, and, consequently, cannot correspond to the definition of a plane.

Strictly speaking, there is but one plane, which may assume three different positions; viz. horizontal, vertical, and inclined.

An horizontal plane is the level ground, or floor; and all planes parallel to the ground or floor are called horizontal planes.

A vertical or upright plane is perpendicular to the ground or floor, as the surfaces of the walls of houses, &c.

An inclined plane inclines to the horizon or to the level ground, in an angle always less than 90^0: the roofs of most houses are inclined planes, the upper parts of writing desks, &c.

Note. A plane may incline or be perpendicular to planes, in many positions, as well as to vertical or horizontal planes.

CONSTRUCTION OF PLANES.

Required the construction of an horizontal plane, 8 feet long by 4 feet wide, $\frac{1}{4}$ of an inch to a foot.

Draw AB, *fig.* 26, indefinitely, make AD perpendicular to AB, lay off 8 feet from A to B, and 4 feet from A to D; with the radius AB, and centre D, describe an arc at C; with the radius AD, and centre B, cross the former arc in C; join CD and CB, the figure ABCD is the plane required.

In this plane all the angles are right angles;

but we cannot represent planes intersecting each other, nor exemplify the construction of solids by planes in this or in similar positions.

Now imagine hinges, or swivels, fixed at the four corners of *fig.* 26, then if this figure be pressed at the corner D, it can be made to assume the form of *fig.* 27; and this form is the most convenient for our present purpose.

Here we may consider all the angles right angles, although not apparently so.

Draw AB, *fig.* 27, make A*y* perpendicular to AB, draw AD so as nearly to bisect the angle *y*AB: make AB equal 8; and AD equal 4 feet, as in *fig.* 26.

Draw DC parallel and equal to AB, draw BC parallel and equal to AD, and *fig.* 26 is put in an oblique position.

Required the construction of the two foregoing planes, and to be made twice the dimensions; that is, the length 16 feet, breadth 8 feet.

To make one Plane perpendicular to another Plane.

Draw AB, *Plate* 4, *fig.* 28, equal to 8 feet, draw AD obliquely, as in the figure, and equal 4 feet; draw DC parallel and equal to AB; draw BC parallel and equal to AD; find E the middle of AB, draw EF parallel to AD or BC;

make EH perpendicular to AB, and equal 4 feet and a half, that is, from a quarter scale, divided into ten equal parts, it will be 4 feet and five tenths, or 4 feet 6 inches; draw FG parallel and equal to EH, join HG, and EFGH is a vertical plane, perpendicular to the horizontal plane ABCD.

Required the foregoing construction double the size of *fig.* 28.

To construct Three Planes, one Horizontal, one Vertical, and one Plane inclined to both.

Draw AB, *fig.* 29, equal 8 feet, as before, AD equal 4 feet, DC parallel and equal to AB; and BC parallel and equal to AD: from *e*, the middle of AB, draw *eh* perpendicular to AB, and equal 4 feet 6 inches; draw *fg* parallel and equal to *eh;* join *hg*, and *efgh* is the vertical plane.

The section or bottom edge BC of the inclined plane BC*gh* might have various positions in the plane ABCD, as well as in BC; join B*h* and C*g*, and BC*gh* is the inclined plane, making an angle with the horizontal plane ABCD, equal 48°, and with the vertical plane *efgh*, equal 42°, the complement of 48°: the angle *he*B being 90°, or a right angle: hence the sum of the three angles is 180°.

Required the foregoing construction to twice

the size of *fig.* 29; and the inclined plane to make an angle of 58°, with the horizontal plane.

Here note it is of little consequence in the required construction, whether the top of the inclined plane be above or below the top of the vertical plane.

To construct Three Planes, viz. one Horizontal and two Vertical Planes, at right Angles to each other.

Construct the horizontal plane A B C D, *fig.* 30, as before; from *i*, the half of A B, raise *i k* perpendicular and equal 4 feet; draw *i h* parallel to A D or B C, make *h m* parallel and equal to *i k*; join *k m*; find the half of B C, and A D; draw *e g*, cutting *i h* in *f*, draw *f l*, *g o*, and *en*, all parallel and equal to *i k*; join *n l o*, and the three planes are constructed as required.

Required the foregoing construction to twice the size of *fig.* 30.

To construct Two Horizontal Planes, two Vertical Planes, and two Inclined Planes; these last two parallel to each other, and one of them above, the other below the horizontal Plane or level Ground.

Construct the horizontal plane A B C D, *Plate* 5, *fig.* 31, as before; from *e*, the middle of A B,

raise *e h* perpendicular and equal 4 feet 6 inches; draw *e f* parallel to A D or to B C, draw *f g*, B *k*, and C *i*, all parallel and equal to *e h*, join *i g* and *k h;* draw A *h*, making an angle of 50° with A B, draw D *g* parallel to A *h*, draw *e m* parallel and equal to A *h*, draw also *f l* parallel and equal to *e m*, join *l m*: here the six planes may clearly be understood by attentively viewing the figure.

Required the foregoing construction to twice the size of *fig.* 31; and the inclined planes to make angles of 40°, with the horizontal planes, instead of 50°.

To construct Two Horizontal Planes, Three Vertical Planes, and Two Inclined Planes, parallel to each other, and making Angles of 26° 30′ *with the Horizon.*

Draw A B C D as before, make *e i* perpendicular to AB, and equal 6 feet, and *e h* equal 4 feet; draw *e f* parallel to A D or B C, draw *f k* equal and parallel to *e i*, make *f g* equal *e h*, join *h g;* draw B *m* and C *l* each equal and parallel to *e h*. Join *m h* and *l g;* make the angle *y m x* equal 26° 30′: draw *m i* and *l k:* make the angle *o e* A equal *y m x*, or, which is the same in effect, draw *o e* parallel to *i m;* also draw *f n* parallel to *o e*, join *o n:* here *e o* is equal *m i:* join *n* D and *o* A; and all the proposed planes are constructed.

Required the above construction twice the size of *fig.* 32, and the inclined planes to make angles with the horizon 35^0, instead of $26^0\ 30'$.

The constructions and intersection of planes are found essentially useful in the study of perspective.

OF SOLIDS.

Solid, from the Latin *solidum*, means a whole, probably because in a solid are contained points, lines, planes, or curved surfaces.

Solids may consist of planes, curved surfaces, or of planes and curves.

At present we will speak of solids consisting of planes.

The principal right lined solids are the cube, the prism, pyramid, &c.

The cube, from the Greek *cubos*, means a figure bounded by squares; for the cube has six faces, or six equal squares.

Prism, from the Greek *prisma*, may mean, a cutting of, or giving a peculiar form to any substance: a prism may have many sides, according to the form of its base, which may consist of a triangle, a square, or any polygon, regular or irregular.

Pyramid, from the Greek *puramis*, means a fire in a state of blazing, in which case, the

flame seems to contract upwards to a point, appearing to diminish as it ascends from the base, or the place of the fuel.

A pyramid may have a triangle, a square, or any polygon, for its base; its sides are all triangles, the least angle of every side meeting at the top, which is called the vertex of the pyramid.

To construct a Cube, as seen on the Left.

Let the given side of the cube be 3 feet, from a quarter inch scale.

Draw AB, *Plate* 6, *fig.* 33, equal 3 feet, raise A*g* perpendicular to AB, and equal to AB; draw B*h* parallel and equal to A*g*; join *gh*, draw BC in any oblique direction toward the right; make BC equal AB, draw *he*, AD, and *gf*, all parallel and equal to BC; join *e*C, CD, *f*D, and *fe*; and the cube is constructed.

Required the construction of two cubes from the same scale, the side of one cube 5 feet, the side of the second 4 feet.

To construct a Cube as placed on the right Hand.

Construct the square AB*de*, *fig.* 34, as in *fig.* 33; draw AD in any oblique direction toward the left, make AD equal to AB, draw BC, *ef*, and *dg*, all parallel and equal to AD,

join DC, D*f*, *fg*, and *g* C, and the cube is constructed as required.

Required the construction of two cubes, of the same dimensions as those given in the last figures; but let these have a similar position to *fig.* 34.

To construct a Cube as seen cornerwise.

Draw any line as *xy*, *fig.* 35, from any point in *xy*, as A, raise A*e* perpendicular to *xy*, and equal 3 feet: bisect the angle *e*A*x*, and also the angle *e*A*y*; through each point of bisection draw AD and AB, make AD and AB each equal A*e*, draw *eh* and *ef* equal and parallel respectively to AD and AB; draw DC and *hg* each equal and parallel to *ef*, or AB; draw BC and *fg* parallel to *eh* or AD: join *h*D, *g c*, and *f*B, and the cube is constructed as proposed.

Required the construction of two cubes, placed as *fig.* 35, and the dimensions the same as proposed in the last example.

To construct a Prism having a square Base.

Draw AB, *fig.* 36, equal 2 feet, from a quarter scale; raise A*g* perpendicular to AB, make A*g* equal 7 feet, draw B*h* parallel and equal to A*g*: join *g h*, draw BC in an oblique direction toward the right, make BC equal AB, draw AD, *h e*, and *gf* parallel and equal to BC; join *f e*,

also *f* D and *e* C; and the prism is constructed as proposed.

Here note that all the four foregoing solids might be constructed by first forming their bases, and raising perpendiculars from each angle, and having determined the altitude, join the points at the top.

Required the construction of two prisms, same scale, one, 3 feet the side of the base, and 10 feet high, the other, 4 feet the side of the base, and 12 feet high, placed alternately as *figs.* 33 and 34.

To construct a Pyramid having a square Base.

Draw AB, *fig.* 37, equal 2 feet, draw BC in an oblique direction toward the right, make BC equal AB, draw AD parallel and equal to AB, join BC.

A Diagonal, from the Greek *dis*, twice, or double, and *gonia*, an angle, that is, meaning a line drawn through two opposite angles.

Now draw the diagonals AC and BD, crossing in *o*, the centre of the base. The altitude or height of a pyramid is laid off on a line perpendicular to the base.

Draw A*x* perpendicular to AB, draw *o e*, the axis of the pyramid, parallel to A*x*, make *o e* equal 8 feet; join *e* A, *e* B, *e* C, and *e* D, and the pyramid is constructed, as proposed. The

altitude might be determined without A x, by letting fall a perpendicular from o on A B, and producing it upward to e.

Required the construction of a pyramid, the side of the base to be 3 feet, from a half inch scale, and the altitude 12 feet.

To construct a Prism, and to place a Pyramid on its upper Surface or Plane.

Draw A B equal to 4 feet, raise A e and B f perpendicular to A B, and equal 1 foot, join ef, draw B C in any oblique direction toward the right, make B C equal A B, draw fg, eh, and A D all parallel and equal to B C; join hg, h D, D C, and g C, and the prism is constructed. Next find k and i, the half of fg, and the half of ef; the base of the pyramid is to be 2 feet: lay off 1 foot from k to p, and from k to q, also from i to n, and from i to o, draw ps, $\overset{k}{4}m$, and qr all parallel to gh, or fe, draw lines in like manner from o, i, and n, all parallel to eh, or fg; these lines crossing on the prism, form the square $vwty$, the base of the pyramid, and determine also its centre x: draw xz parallel to A e, or to D f; lay off 8 feet from x to z, join zv, zw, zt, and zy, and the proposed figure is constructed.

Required the construction of a similar figure twice the dimensions of *fig.* 38.

PRELIMINARY OBSERVATIONS

ON

PERSPECTIVE.

THERE are two very important terms used in the study of Perspective; and these are, a vanishing line and a vanishing point.

As the whole art of perspective depends on clearly understanding these terms, we will attempt an explanation, which, we trust, may prove satisfactory.

VANISHING LINE.

Let us imagine a perfectly level horizon: let us also imagine a road twenty feet wide, and one end of it produced to where the sky seems to meet the land; the remote end of this road, we say, is at an infinite distance.

At the near end of this road, imagine a

plane of glass, of infinite extent, placed perpendicularly on the road, so that the sides of the road may be perpendicular to the plane of the glass.

Now imagine a spectator placed at the distance of twenty feet from the glass, and in the centre of the road; and let the height of the spectator's eye be five feet.

If the spectator look through the glass toward the remote end of the road, this end appears contracted to a point, and seems to rise up to the level of his eye: hence the whole road assumes a triangular form, the base of which is the intersection of the glass with the road; and the vertex of this triangle vanishes or disappears, at the remote end, to the spectator's view.

Now, as the distant extremity of the road vanishes in a point, where the sky seems to meet the land, we may properly call this remote extremity of the horizon the vanishing line in nature.

But, as the spectator means to represent this road in perspective on the glass, he must also have a perspective vanishing line, and this is obtained in the following manner:

Imagine a plane, of infinite length, placed horizontally with one edge at the spectator's

eye, and this edge parallel to the plane of the glass; in this position this horizontal plane will be five feet high.

Imagine this plane to move in a horizontal direction, cutting the glass, and produced to the remote extremity of the road, where it will seem to coincide with the vanishing line in nature.

In the progress of this horizontal plane, it cut the glass at the height of five feet, and produced the required perspective vanishing line, which, being parallel to the horizon, we call the horizontal vanishing line on the glass, or on the picture.

From this theory we may conclude that the perspective vanishing line is parallel to the vanishing line in nature; and if produced or moved parallel to itself, they would both coincide at an infinite distance.

VANISHING POINT.

All things remaining as before, imagine a line drawn from the centre of the intersection of the glass with the road; and this line produced along the middle of the road to the vanishing line in nature, its extreme end will vanish or disappear in that point, in which the remote extremity of the road vanished.

Now this line, as well as each side of the road, is supposed perpendicular to the glass or to the picture, and all vanish in one point, in the vanishing line in nature, which point is directly opposite the spectator's eye.

To find a corresponding perspective vanishing point on the glass or picture, imagine a line drawn from the eye of the spectator parallel to the line drawn along the middle of the road, and the former will vanish in the same vanishing point in nature as the latter: but the former line drawn from the eye, in its progress cut the glass or picture in a point, and this point is the perspective vanishing point on the picture for all lines perpendicular to the picture or glass.

We call the road an original object, to distinguish it from its perspective representation.

We also say the road is an horizontal plane, and its vanishing line on the glass or picture is that line made by the plane which passed through the eye of the spectator parallel to the horizon.

A line drawn from the spectator's eye perpendicular to the plane of the glass or picture, is called the centre of the picture; but the line drawn from the spectator's eye parallel to the line drawn along the middle of the road, is perpendicular to the plane of the picture, and

the point in which it cut the picture is the centre of the picture: hence the centre of the picture is the vanishing point for all lines perpendicular to the plane of the picture.

Having now the vanishing line of the road, and the vanishing point for the sides of the road, its perspective representation is easily found, thus: from each extremity of the line or section made by the glass and the road, draw a line to the centre of the picture; and these two lines form a triangle, the base of which is the section of the picture with the road: and the vertex in the centre of the picture and this triangle we call the perspective representation of the road.

We are aware that the introduction of new names into any art or science may often embarrass a learner; wherefore I beg the indulgence of my readers, if, from long perusing the science of perspective, I find it indispensable to use a name of which I have never heard before.

In the foregoing discourse we have spoken of the horizontal vanishing line in nature, as being at an infinite distance from the spectator. Now, if we suppose a plane perpendicular to the horizon, to pass through the vanishing line in nature, and this plane to be infinitely ex-

tended, we may call this plane the plane of infinite distance: and, in this plane, let us suppose all original planes, however opposed to the picture, to vanish.

To exemplify this hypothesis, let us suppose the common roof of a house, as placed before the glass or picture, so that the ridge or top of the roof may be parallel to the plane of the picture. Here each side of the roof, that is, each plane forming the side of the roof, is inclined to the picture, and also to the horizon.

Now, if we imagine the side of the roof next the picture to be produced upward toward the sky, it will cut the plane of infinite distance, and determine a vanishing line in nature.

And if we imagine a plane to pass from the eye of the spectator parallel to the same side of the roof, and produced, it will seem to coincide with the forementioned vanishing line in nature, and, in its progress, will cut the picture, and determine the perspective vanishing line for the near side of the roof.

Again, if we imagine the remote side of the roof to be produced below the horizon, it will cut the plane of infinite distance, and determine its vanishing line in nature: and a plane passing from the eye of the spectator parallel to the same side of the roof will seem to coin-

cide also with the foregoing vanishing line in nature, and, in its progress, will cut the picture, and determine the perspective vanishing line for the remote side of the roof below the horizontal line on the glass.

Here the former vanishing line in nature is *above* the horizontal vanishing line in nature, and the latter vanishing line in nature *below* the horizontal vanishing line in nature: the case is similar with regard to the perspective vanishing lines on the picture for the sides of the foregoing roof.

In the foregoing remarks we have spoken of a vanishing line above, and of another below, the horizontal vanishing line on the glass or picture, as it is called; but it frequently happens that original planes are so opposed to the picture and to the horizon, as to require vanishing lines intersecting other vanishing lines in various angles of inclination, all which tend to prove the invariable analogy between the plane of the picture, and the imaginary plane of infinite distance.

To the study of the construction of right lined solids, I have devoted much of early life: I have accordingly introduced the geometrical construction of many of the solids previous to their perspective representation, which cannot

but simplify the practice of perspective to beginners, as I have often experienced.

As I have, in my practice of teaching, been obliged to use the language of patient repetition, so I have adopted a similar mode of proceeding in this little work; which I hope they who are well acquainted with the science will excuse, as I write for those who have never studied the subject.

PERSPECTIVE.

A TRANSPARENT substance or body is that through which we can distinctly see objects on the opposite side: water, glass, &c. are transparent bodies.

An opaque body is that through which we cannot see, as iron, wood in general, &c.

Perspective is a compounded Latin word, from *per*, through, and *specio*, to view, that is, to view objects through a transparent substance.

If, at the distance of 12 or 18 inches, I look through a pane of glass, at a window in a house opposite, and, keeping my head steady, I make on the glass, with a little brush of dark colour, the four corners of the window, just as they appear to my eye, then join the points on the glass, this squarelike form will be the perspective representation of the opposite window.

In the above example, the eye being fixed, and lines imagined to be drawn from the four

corners of the window to the eye, these imaginary lines, and the window opposite, form a figure like a pyramid, the window opposite being the base, and the vertex of the pyramid in the eye. The pane of glass through which I look is a plane cutting this imaginary pyramid, and the square-like form on the glass represents its original, which is the window opposite.

OF THE ANGLE OF VISION.

The principal parts of the eye for our purpose are the pupil and the retina.

Pupil, from the Latin *pupilla*, probably means a diminutive figure of a person, as the representation of a little human figure may often be seen in the pupil of the eye.

Retina, from the Latin, means a net, or any thing like network, because this part of the eye has the appearance of network.

If I look at a point, a ray of light comes from that point, and passes through the pupil of my eye, and is transmitted directly to the retina, and thence to the brain.

The retina is of a limited round magnitude, so that a ray may enter the pupil, but that ray may not fall on the retina; for it may pass above it, below it, or on one side, and, in such cases, there can be no perfect vision; hence, it

has been ascertained, that one or many objects should be viewed under a certain angle.

Let the corner of a house, considered as a straight line 20 feet high, be used to exemplify this theory of vision.

If a person stand within 5 feet of this corner, he cannot view the whole height distinctly, because a line drawn from the top of the corner, and passing through the pupil, falls below the retina; and a line drawn from the bottom, passing, in like manner, through the pupil, ascends above the retina; hence the corner of the house cannot be distinctly seen, because the distance is too near or too short.

But, if the spectator remove from the corner until he can with ease view each extremity of the corner, the whole line will fall on the retina, and produce distinct vision.

Now the distance of the spectator from the corner of the house will be found to be about 20 feet: hence is derived a general rule for viewing large objects, which is, that a person, to view an object, must make his distance equal the greatest dimension of the object, which is usually under an angle of 53^0 or 54^0.

For a very minute object, a perfect eye cannot view it distinctly, at a distance less than 7 or 8 inches.

OF THE HORIZON.

Horizon, from the Greek *horizo*, means the limits of vision.

We may consider three kinds of horizons, viz. the rational horizon, the sensible horizon, and the perspective horizon.

By the two first horizons we mean planes having a circular boundary.

The rational horizon is supposed a plane cutting or dividing the whole earth into two equal parts, called hemispheres: a spectator is supposed standing on the top of one hemisphere, and the cutting plane parallel to the ground on which he stands, supposing the ground level.

The diameter of the earth is nearly eight thousand miles, the half is nearly four thousand, hence the distance from the spectator to the rational horizon, is nearly four thousand miles.

The limit of the sensible horizon is the distance from the spectator at which he can see objects distinctly, which is about three miles and a half.

The perspective horizon is quite indefinite, it means that distance from the spectator at which the sky seems to meet the land.

Construction of Fig. 39, *Plate* 7.

This figure is constructed from a scale of one-tenth of an inch to a foot; hence every inch contains ten feet; this is the scale in our future operations.

It is requested the pupil may have neat instruments, and the best pencils.

Draw A B, lay off 25 feet from A to B, draw B C in any oblique direction toward the right, make B C equal 10 feet, draw A D parallel and equal to B C, join D C.

Make A I equal 15 feet, and draw I T parallel to B C, raise I *u* perpendicular to A B, and equal 13 feet; draw T *v* parallel and equal to I *u*, join *u v*.

Find *f* the half of B C, and draw *f a* parallel to A B, draw *b s* parallel to I *u*, draw *f e* equal 5 feet, and *a k* parallel to I *u*; join *e k*, cutting *b s* in *o*; draw *m o n*, produced both ways, P *e q* and *y k z* all parallel to B C, or A D.

Draw B P and C *q* parallel to *f e*; join *q n* and P *m*, draw A *w* and D *x* parallel and equal to I *u*; join *x v* and *w u*; join *v k*, *u k*, T *k*, and I *k*, make I L equal 5 feet, draw I *o*, draw also L *e*, cutting I *o* in the point *l*.

Make *o r*, *o h*, and *o s* equal to *o e*; produce *u* I toward *g*, and produce T I toward G; make

I *g* and I G each equal to I L, draw *g s*, cutting I *o* in the point I, draw G *r*, cutting I *o* also in the point I, and the figure is constructed.

Explanation of Fig. 39.

The pupil is requested to attend particularly to the explanation of this figure, as it probably contains the rudiments of perspective.

Before we explain *fig.* 39, let us premise the following theory.

Imagine a street 40 feet wide, the houses all equally high, and the street so long that the remote end may seem to become almost invisible.

If a spectator view this street from one end, and stand about 40 feet distant, and opposite the middle of the street. In this position the near end of the street will seem in its original dimensions, but the remote end will seem to rise to a level with the spectator's eye, and contract to a point assuming a triangular form, and this point will be the vertex of the triangle.

The tops of the houses also, on each side, will seem to come down, or grow lower, until they vanish or disappear like the remote ground, and all in the same point.

Now if a line be drawn through this point parallel to the horizon, and to the end of the

street, this may be called the vanishing line in nature, that is, the horizontal vanishing line.

Hence we may infer, that the horizontal vanishing line precisely separates the land from the sky.

A B C D, *fig.* 39, is called an original plane, by some called the ground plane.

I *u v* T, the picture on which A I T D, part of the original plane, is to be represented.

f e, the spectator, five feet high from *f* to *e*, and *e* his eye.

f, his feet, and called the station point.

o, the centre of the picture, because *e o* is perpendicular to the picture, *o* is also the vanishing point for all lines and planes perpendicular to the picture.

h o r, the horizontal vanishing line on the picture.

A D, the remote extremity of the ground.

Now let *e* be the eye of the spectator, and *f* his feet.

Let A I T D be the ground of a street, I *u v* T a large square of glass, it is required to show the representation A I T D on the glass or the picture.

A D being at an infinite distance, rises as high as *y z*, where the sky seems to meet the land A D; it also contracts to a point as in *k*, and *a k* is equal *f e*, the height of the spectator's eye.

Join I k and T k, and the triangle I k T is, in nature, the appearance of I A D T. Now let this triangle be represented on I u v T.

The line e k drawn from e, the eye, to k, the vanishing point in nature, cuts the picture in o, the vanishing point on the picture for all lines perpendicular to the picture; and h r being drawn through o, parallel to y k z, is the horizontal vanishing line on the picture.

Here I T, the near end of the street, touches or cuts the picture; now k is the vanishing point in nature, and o the vanishing point on the picture; draw I o, and I o is the perspective of I k; draw T o, and T o is the perspective of T k; hence the triangle I k T is represented by the perspective triangle I o T on the picture.

Let A w u I and D x v T, be the two sides of a street, considered here only as planes, these sides will respectively assume the forms of the triangles I k u and T k v: on the picture draw v o and u o, and I o u will be the perspective of I k u, and T o v will be the perspective of T k v.

Again, suppose this street to be covered over on the top, and let the plane u w x v represent this cover, it will assume the form of the triangle u k v, and u o v will be the perspective representation of u k v, but u k v is the appearance in nature of u w x v, therefore u o v, on the

picture, is the perspective representation of *u w x v;* and the same may be proved of the ground, and the two sides of the street.

It is evident, from viewing the four planes constituting the ground, sides, and top of the street, that they are perpendicular to the picture; and the extreme or remote ends vanish in the centre of the picture; hence we obtain the theorem, that all lines and planes perpendicular to the picture vanish in the centre of the picture.

To find a Vanishing Line and Vanishing Point by Brook Taylor's Rules.

To find or determine a Vanishing Line on the Picture, for any original Plane.

Rule.—Imagine a plane to pass through the eye of the spectator parallel to the original plane, and where the plane so produced cuts the picture, determines the vanishing line.

Let A D T I, *fig.* 39, be an original plane (a plane before represented in perspective is called an original plane).

Let *m n q* P be a plane parallel to A D T I, which here cuts the picture in *m n*, the vanishing line required; for the original plane A D T I.

Having found the Vanishing Line for any Plane, to find the Vanishing Point for any Line in that Plane.

RULE.—Draw a line from the eye parallel to the original line, and where this line cuts the vanishing line of the plane, in which the original line is, determines the vanishing point.

Let A D T I be an original plane, and *h o r* its vanishing line, L I is a line in that plane, to find its vanishing point on the picture.

Draw *e o* parallel to L I, and it cuts *h r* in *o*, the vanishing point required.

To put L I *in Perspective, a Part of the Side* I A.

L I cuts or meets the picture in I, then because L I is perpendicular to the picture, I is called its seat on the picture: draw a line from I, the seat, to *o*, its vanishing point, and I *o* is called the indefinite representation of L I; draw L *e*, cutting I *o* in *l*, and I *l* is the perspective representation of L I.

From the effect of the foregoing rules may be seen the same effect produced by considering our explanation of a long street; hence the rules of Brook Taylor may be relied on, which

are generally, in all cases, an evident proof of this great man's genius.

Although *fig.* 39 serves to render intelligible the principal difficulties in perspective, yet it presents a distorted appearance of objects on the picture, for which reason *fig.* 40 is usually preferred.

To construct Fig. 40.

Draw D A equal D A in *fig.* 39; draw A *p* in *fig.* 40, perpendicular to D A; draw D *q* equal and parallel to A *p*, join *q p*.

Make A I equal A I in *fig.* 39; draw I T, produced both ways in *fig.* 40, parallel to D A.

Make I *m* equal I *m* in *fig.* 39, and draw *m n* produced both ways, *fig.* 40. Make I *u* equal I *u* in *fig.* 39; join *v u*.

Make I B equal I B in *fig.* 39, and draw B C, *fig.* 40; draw *o e* parallel to A P, and equal to *o e* in *fig.* 39.

Make *o r* and *o h* each equal to *o e*, make I L equal to I L, *fig.* 39; make I G and I L each equal to I L, *fig.* 40.

Draw I *o* and T *o;* draw L *e*, cutting I *o* in *l;* draw 3 *h*, cutting I *o* in *l;* draw G *r*, cutting I *o* in *l*.

Find *k*, the half of D A, and draw I *k* and T *k*, and *fig.* 40 is constructed.

ON PROJECTIONS.

Projection, from the Latin *projicio*, to throw or cast, may have various significations, but, confined to our operations, it means the appearance of a point, line, plane, or solid, on a plane, determined by lines drawn from the object perpendicular to the plane on which the projection is to be made.

If I hold a shilling between my eye and a wall, or between my eye and the floor, so that the plane of the shilling may be parallel to the plane of the floor, and imagine lines drawn from the circumference of the shilling perpendicular to the floor, the projection on the floor will be a circle; this is also called the seat of the shilling on the floor.

If I hold the shilling obliquely, that is, the plane of the shilling oblique to the plane of the floor, and lines drawn from the shilling as before, the projection, or seat of the shilling on the floor, will be an ellipse, or oval.

If I hold the shilling edgewise, that is, the plane of the shilling perpendicular to the floor, and lines drawn as before, the projection, or seat of the shilling on the floor, will be a straight line, equal to the diameter of the shilling.

The first is called a direct view, the second an oblique view, and the third a right view.

Imagine a swivel at IT, *Plate* 7, *fig.* 39, at BC, at P*q*, at *mn*, and at *fe;* now imagine the picture I*uv*T to be pressed forward, so as to lie flat, then *fig.* 39 will represent *fig.* 40, as may be seen by comparing the respective points, lines, and planes in the one with those in the other.

In this case the plane *uwxv*, in *fig.* 39, is projected into the right line *vw*, in *fig.* 40; and the elevated triangle I*k*T, in *fig.* 39, is projected into the triangle T*k*I, in *fig.* 40.

In *fig.* 39, the spectator is standing as represented by *fe*, but in *fig.* 40 he is supposed on his back.

The distorted angle I*o*T, in *fig.* 39, is the perspective representation of the original plane ADTI, which is all the ground beyond the picture; and the angular point at *o*, on the picture, represents the vanishing point of the original line AD, in the plane ADTI.

The horizontal line *hor* represents that line in nature that separates the sky from the land, and its altitude from the ground is always equal to the height of the spectator's eye, from the ground, or from the horizon.

The foregoing lines and points in *fig.* 39 have a more pleasing appearance in *fig.* 40.

It is common in perspective, for the sake of convenience, to transfer or transpose the eye from one point to another.

Let us suppose the point L, *fig.* 39, in the side I A, required to be put in perspective, that is, represented on the picture; as it would appear to an eye at *e;* here, as in all future operations, we consider the picture transparent.

Now the seat of the point L, on the picture, is I; draw a line from the seat to the vanishing point of L I, and this is *o;* because L I is perpendicular to the picture.

I *o* is the perspective of I A produced infinitely, but the point L is in I A, and must therefore appear in I *o;* draw L *e*, cutting I *o* in *l*, and *l* is the perspective representation of the original point L.

Let *e*, the eye, be transposed in *r*, in order to put the point L in perspective, make *o r*, *o h*, and *o s*, each equal *o e;* now suppose the eye at *r*, it is evidently in the plane of the picture, and if a line be drawn from L, the original point, it will not cut I *o*, because L is not in the same plane, viz. the plane of the picture; here we must imagine the picture produced to the left; produce T I toward G, make L G equal I L, and the point L is transferred to G, draw G *r*, cutting I *o* in *l*, as before.

In like manner, if we transfer the eye to *h*, and transfer the given point L, in *fig.* 40, it is transferred to 3, draw 3 *h*, cutting 1 *o* in *l*, as before.

Finally, suppose the eye transferred to *s*, *fig.* 39, here also the eye is in the plane of the picture, but the picture must be produced downwards, as in *g*, make 1 *g* equal 1 L, draw *g s*, cutting 1 *o* in *l*, as in the preceding examples, these are also exemplified in *fig.* 40, more pleasing to view.

A line drawn from the eye, perpendicular to the plane of the picture, determines the centre of the picture.

In constructions like *fig.* 39 this appears not easily performed, but may be simply done thus: *f*, is the feet or station point, from *f* produce *f b*, perpendicular to 1 T, draw *b s* perpendicular to 1 T, or parallel to 1 *u*, let fall a perpendicular from *e*, the eye, on *b s*, cutting *b s* in *o*, the centre of the picture required.

This method will determine the centre of the picture, whether upright, inclined, or reclined. Because *b s* is in the plane *u v* T 1 and *e o* perpendicular to *b s*, the line *e o* must also be perpendicular to the plane *u v* T 1.

Here *o* is the centre of the picture; *e*, *r*, *h*, *s*, respectively, the places of the eye; and *o e*, or

bf, the distance of the eye from the picture. Wherever the eye is placed it is called the point of sight, and thus may be seen the error in calling the centre of the picture the point of sight.

QUESTIONS FOR PRACTICE.

How many kinds of planes are there?

What is a vertical plane?

What is an horizontal plane?

What is an inclined plane?

What is the difference between a vertical plane and an horizontal plane?

What is the difference between an inclined plane and a vertical plane?

How is the inclination of one plane to another measured?

If two planes intersect, what kind of a line does their intersection make?

Why are the intersections of planes always right lines?

What kind of planes are floors and flat ceilings?

What kind of planes are the common walls of a room?

What kind of planes are the common roofs of houses?

Suppose an inclined plane to cut an horizontal plane, and a vertical plane, the angle of the inclined plane with the horizontal plane is 35^0 41′, required its angle with the vertical plane, all their intersections being parallel?

How many planes in a cube, and what kind are they?

How many planes in a pyramid, and what kind are they?

Is the surface of a hill a plane?

What is the difference between a surface and a plane?

What is perspective?

How may perspective be exemplified?

What is the angle of vision?

What is the pupil of the eye?

What is the retina?

If I stand at 15 feet distance, to view a house 20 feet high, by 30 feet long, can I view it distinctly? If not, at what distance should I stand?

What is the horizon?

What is the rational horizon?

What is the distance of any spectator from the plane of the rational horizon?

What is the sensible horizon?

Of what diameter is the sensible horizon?

What is the distance between the sensible and the rational horizon?

What is that horizon called which limits our view to where the sky seems to meet the land?

Required the construction of *fig.* 39, *Plate* 7, to 3-tenths of an inch to a foot, in which case the required figure will be three times larger.

What is the horizontal line in nature?

What is the vanishing point in nature?

What is the picture?

What is an original plane?

What is the horizontal vanishing line on the picture?

Which is the vanishing point, on the picture, of lines and planes perpendicular to the picture?

What is Brook Taylor's rule to find the vanishing line of an original plane?

What is his rule to find the vanishing point for any original line?

Required the construction of *fig.* 40, *Plate* 7, to a scale of 3-tenths of an inch to a foot.

In what position must a ring be supposed, to make its projection on a plane, a circle?

How should I hold it to make its projection on a plane, an ellipse?

If I wish to make its projection on a plane a right line, how must I hold it?

Is the centre of the picture the point of sight?

Which is the point of sight?

Plate 8, *fig.* 41. This figure is constructed like *fig.* 40, *Plate* 7; here *m n* is the width of the picture, the part above the horizontal line is omitted, being unnecessary; *m n* here is wider than in *fig.* 40, *Plate* 7, this being 15 feet, taken from a scale of 1-tenth of an inch to a foot, as before.

The distance of the eye from the picture, that is, from the horizontal line, must always be the breadth of all the objects, or of a single object, when given to be represented in perspective; so here, *o e*, the distance of the eye, is equal to *s v*, the whole breadth of all the original objects.

In *fig.* 41 we have lines and squares to represent in perspective.

Here we may take any original line, as D E, the space between the two squares A B C D and E F G H.

We shall use Brook Taylor's rules.

Find the seat of the original object on the picture, draw a line from the eye parallel to the original object or line, and where it cuts the vanishing line of the plane in which the original object is, determines the vanishing point. This is peculiarly applied to a line.

Draw a line from its seat to its vanishing point, and this will be its indefinite representation.

Draw a line or lines from the original object to the eye; where this line or these lines cut the indefinite representation, determines the perspective of the original object.

The seat of D E is *t*, and whether D E be 5 inches or 50 feet, the point *t* is its seat, because D E is projected into a point, which is *t*.

e o is parallel to D E, hence *o* is its vanishing point; draw *t o*, and *t o* is the indefinite representation of D E; draw to *e*, the eye, D *e*, cutting *t o* in *d*; draw E *e*, cutting *t o* in *é*, and *é d* is the perspective of D E.

When we use similar letters in reference, a mark is put over one letter to distinguish it from a similar one; thus, *é* in *é d*, is distinguished from *e*, at the place of the eye.

The line *e o*, produced downward or upward, is sometimes called the prime vertical.

To put k l in Perspective.

Here, as in D E, *o* is the vanishing point for *k l*, *i* is the seat of *k l*, and *i o* its indefinite representation; draw *k e* and *l e*; here the indefinite representation, and the two lines drawn to the eye, form one line, as *e o i k l*, wherefore *k l* cannot be put in perspective by the method used in D E, between the squares, on the right.

Transfer *e*, the eye, to *h*, by making *o h* equal

o e; transfer also the original line *k l* to the picture, by making *i r* equal *i k*, and *i p* equal *i l*, draw lines from *r* and *p* to *h*, cutting *i o*, and *y x* is the perspective of *k l*, as proposed.

Again, let A B, one side of the square A B C D, be required in perspective.

A B, like the foregoing lines, is in the horizon, and the vanishing line is *r o h*, as before.

Now if a line be drawn through *e*, the eye, parallel to the original line A B, it can never cut *r o h*, the vanishing line of the plane in which A B is; wherefore, the vanishing point for A B cannot be obtained, because A B is parallel to the picture, but if we find the seats, and the perspective of the extremities of any original right line, we may join such extremities, and the perspective of the original line is obtained.

The seats of the points A and B, and, consequently, of the whole line A B, may be considered *t s*, and as A *t* and B *s* are perpendicular to the picture, they will vanish in *o;* draw *s o* and *t o*, draw A *e*, cutting *t o* in *a*, one point; here A B is parallel to the picture, and its perspective will also be parallel; draw *a b* parallel to the intersecting line *v w*, to cut *s o*, and *a b* is the perspective of A B. This might be found by drawing a line from B to *e*, the eye.

To put Two Squares in Perspective.

From *s* draw *s*G perpendicular to *vw*, make *st* equal 4 feet, 1-tenth of an inch to a foot being the scale; draw *t*H parallel to *s*G, lay off the extent *st* four times from *s* to G; draw BA, CD, FE, and GH all parallel to *vw*, and ABCD and EFGH are the original squares.

These two squares are within the parallelogram H*ts*G, which, if produced to an infinite distance, would assume the form of a triangle, as spoken of in our remarks on a long street.

The seat of the two squares, or of a hundred similarly situated, is *ts*, from *t* and *s* draw lines to *o*, their vanishing point, and *tos* is the indefinite space within which the squares will appear.

From A, D, E, H draw lines to *e*, the eye, cutting *to* respectively in *a*, *d*, *e'*, *h*; draw *ab*, *dc*, *e'f*, and *hg* all parallel to *vw*, and *abcd* and *efgh* are the perspective of the original squares.

To put a Vertical Plane, 12 *Feet long by* 4 *Feet wide in Perspective, and let this Plane cut the Picture at right Angles, and the Horizon at right Angles.*

Construct *fig.* 42, as 41, the dimensions being the same.

Draw AD, *fig.* 43, draw AB in an oblique direction toward the right, make AD equal 4 feet, and AB equal 12 feet, draw DC parallel and equal AB, join CB.

From any point in TI, *fig.* 42, as B, draw BA perpendicular to TI, and equal AB, *fig.* 43; produce AB upward to *d*, make B*d* equal AD, *fig.* 43. BA is the plan of AB, *fig.* 43.

Here, because B*d* cuts the picture, it keeps its original height.

This plane being perpendicular to the picture, its top and bottom edges will vanish in *o;* draw B*o* and *do*, draw A*e*, cutting B*o* in *a*, the perspective of A in BA; draw *ac* parallel to B*d*, and B*dca* is the perspective of the original plane.

Let BA, on the right hand, be the plan of another plane 12 feet long, and 8 feet high.

Produce AB upward to *f*, make B*f* equal 8 feet, draw B*o* and *fo;* from A draw A*e*, cutting

B *o* in *h*, draw *h g* parallel to B *f*, and B *f g h* is the perspective required of the given original plane.

Plate 9, *Figs.* 44, 45, *and* 46.

Construct *fig.* 45 by the methods used in constructing *figs.* 41 and 42 of *Plate* 8.

Construct the square A B C D, *fig.* 46, each side 4 feet.

Let this square be considered as a vertical plane placed parallel to the picture, and at the distance of 4 feet; required its perspective, according to these conditions.

x G is the width of the picture.

From *x* draw *x* A perpendicular to *x* G, produced also above *x* G toward *r*, make *x* A and *x r* equal 4 feet.

Make *x y* equal 4 feet, through *y* draw *i* B parallel and equal to *r* A, join A B and *r i*, and A B is the plan of *fig.* 46.

Now, if we imagine the plane of *fig.* 46 to be placed vertically on the plan A B *fig.* 45, and imagine lines drawn from its four corners perpendicular to the picture, they will determine the seat, *x r i y*, on the picture.

If the original square coincided with the picture, *fig.* 46 would represent it, in which case none of its parts would be diminished,

but, in the present case, it will be less than its original size, because removed 4 feet from the picture, as may be seen in the plan AB.

From the four corners of the seat *r*, *i*, *y*, *x* draw lines to *o*, the vanishing point, as *ro*, *io*, *yo*, and *xo;* and here we have the figure of a pyramid, whose base is the seat, and vertex the vanishing point *o:* a section of this pyramid parallel to the seat, will represent the original plane in perspective.

From B, one end of the plan, draw to *e*, the eye, a line which cuts *yo* in *b;* and as the space between *x*G and *mn* is all ground, or the horizon, the perspective of the original plane must be vertical, that is, perpendicular to the ground.

Draw *ba* parallel to *x*G; draw *bu* parallel to *xr*, cutting *io* in *u;* draw *al*, cutting *ro* in *l*, join *lu;* and *abul* is the perspective of the original vertical plane.

To put Two Cubes, and a Prism with a square Base, in Perspective.

Make the square VUTW, *fig.* 44, as in *fig.* 46, draw VH in any oblique direction toward the left, and WG parallel to it; lay off the extent VW seven times from V to H. The respective spaces between each cube and the prism is 8 feet.

Draw XY, NO, QP, EF, and HG, all parallel

to VW; from all the angles of the bases raise perpendiculars, that is, draw lines parallel to VU or TW, complete the cubes as in the figure, make the prism twelve feet high.

Take G, *fig.* 45, the limit of the picture on the right, and through G, perpendicular to *x*G, draw the line 12W; make GH equal 4 feet, draw through H, HV parallel and equal GW; make G*b* equal four feet, and complete the square H*ab*G, which is the seat of the cubes: the three plans are the same as the bases of *fig.* 44.

Put the plans in perspective as in *fig.* 45, from *ab*GH, the seat of the cubes, draw lines to *o*, the vanishing point, which will form a pyramid, as on the left side of the picture.

From all the corners of the perspective plans, viz. from *e f*, *qp*, *no*, *xy* (here the remotest side is too contracted to be lettered), draw perpendiculars, cutting *ao* and *bo*, in *d*, *c*, *r*, *s*, &c. the prism in the middle is thus represented.

Produce *pk*, *qi*, *o l*, and *nm* indefinitely upward.

The seat of this prism in the picture is not shown, as it might cause confusion.

The left and the right hand sides of the cubes are in planes; and hence the vertical line G12, is in the right hand plane of the cubes and of the prism.

The intersecting line xG is called a line of measures for objects in the horizon, and, in the present case, the line G 12 is called a line of measures for any object, or part of an object, in the vertical plane, in which the right hand sides of the cubes and of the prism are placed.

Now draw 12o, and it cuts pk and $o'l$ in k and in l; draw ki parallel to mn, or to xG, cutting qi in i; draw io, cutting nm in m, join ml, which determines the top of the prism; and because the height of the prism is 12 feet, and the height of the eye is 5 feet, the prism is 7 feet higher than the eye, consequently the top cannot be seen.

Here all the remote parts of the original objects appear toward o, the centre of the picture, because the point o represents the extreme part of the horizon, at an infinite distance.

In all perspective drawings we see objects on the picture, in places directly contrary to those places, in which the spectator beyond the picture, is supposed to view them; but if we turn the drawing to the light, and look at the back of it, we shall see objects in their true perspective places; and if the lower part of the paper be turned forward from the intersecting or ground line, so as to become perpendicular to the picture, and to suppose the original objects on their plans, then turn back the upper

part of the paper, from the horizontal line, and perpendicular also to the picture, a gratifying appearance will be exhibited to the student in perspective.

This experiment will answer best for objects below the horizontal line.

To represent Pyramids in Perspective.

Draw FI, *Plate* 10, *fig.* 47, equal 4 feet, draw IN in an oblique direction toward the right, lay off the extent FI four times from I to N, here are 8 feet distance between the bases of the pyramids; draw FM parallel and equal to IN, join MN. The base of each pyramid is 4 feet square, and may be constructed as in the figures; draw diagonals in each base, which determine the points *o*, P their centres.

From C, the half of FI, draw CA, perpendicular to FI, and equal 12 feet, then from *o* and P, the centres of the bases, draw *o*E and PK parallel to CA, draw CD parallel to IN, draw AB parallel to CD, draw CE and QK, join BD.

Here CABD is a vertical plane passing through the centres of the pyramids, cutting one in the line CE, the second in QK: this plane determines their altitude, which is 12 feet. The slant height CE is not the height of the pyramid; but the length of its axis *o*E in this kind of pyramid is the true height.

M*n*, *fig.* 49, the limits of the picture.

Draw MF perpendicular to M*n*, and equal MF, *fig.* 47; make MN equal 4 feet, draw NI parallel and equal MF, complete the squares of the plans as in the figure.

Draw N*o*, M*o*, and *xo*, and the bases will appear within the triangle M*o*N, draw to *e*, the eye, the lines O*e*, H*e*, I*e*, each cutting respectively the line N*o* in *o'*, h, and *i*; draw from *o'*, *h*, and *i* lines cutting M*o*, the left hand side of the bases; draw the dotted diagonals as in the figure, raise *xy* perpendicular to M*n*, which here may be done by producing C*x* upward, make *xy* equal 12 feet, and draw *yo*; from the perspective centres of the bases of the pyramids, draw *pk* and *oe'*, limited by *yo*; draw *k*N, *k*M, *kl*, and *ko*, and one pyramid is completed: draw *e'h*, *e'i*, and two lines to *e'* from the other two corners of the base on the left, which complete the second pyramid.

In *fig.* 47 the plane CABD determined the height of the pyramids; in *fig.* 49 the plane *xyo* determines the height of the perspective pyramids.

Let E*o*, *fig.* 48, be a given line, to construct an inverted pyramid; here E*o* is the axis or altitude of the required pyramid.

Through *o* draw a line parallel to the top of

the paper, as *xy*, draw an oblique line through *o*, inclining to the left, as *w v*.

Make *ow*, *ov*, *ox*, and *oy*, each 2 feet; through *v* and *w* draw AB and DC parallel to *xy*; through *x* and *y* draw AD and BC, cutting the former lines in A, B, C, D; join AE, DE, CE, and BE, and the inverted pyramid is constructed.

This construction may be proved thus:

Draw XY parallel to *xy*, draw VW parallel to *vw*, and make their distances from E respectively equal; through X and Y draw *ad* and *bc* parallel to VW; through V, and through W draw *ab* and *dc* parallel to XY; join A*a*, B*b*, C*c*, and D*d*, all parallel to the axis E*o*, and the figure is a prism with equal bases.

To represent Fig. 48 *in Perspective.*

From *nm*, *fig.* 49, the right side of the picture, draw *n*C perpendicular to *mn*, and equal 4 feet; complete the square as in the figure; draw the diagonals D*n* and MC, crossing in E, draw *mo* and *no*.

Here a line drawn from D to *e*, the eye, would cut *mo* in *x*; but we shall use another method.

D*n* is a diagonal of that square which cuts the intersecting line M*n* in *n*; draw from *e*, the

eye, a line parallel to D*n*, and it cuts the horizontal line in *r*, as *er*.

r, once the transposed place of the eye, is now a vanishing point; draw *nr*, cutting *mo* in *x;* draw *xy* parallel to *mn;* and draw from *e*, the eye, *eh* parallel to the other diagonal *m*C, and *h* is the vanishing point for *m*C; draw *mh*, cutting *xn*, the other perspective diagonal, in *e'*.

The angles formed at the centre of a square by drawing two diagonals are always right angles; hence *m*E*n*, in the original square, is a right angle, or 90°; *er* and *eh* are their respective parallels, and the angle formed at *e*, the eye, is a right angle; and hence we say the angle at the eye, formed by any two lines, is always equal to the angle formed by two original lines, in an original plane.

The inverted base of the pyramid could be determined perspectively, by a method analogous to the construction in *fig.* 48, but the following is more easy for the pupil.

From *nm* and *x* raise perpendiculars; make *ma* and *nb* each equal 12 feet, and join *ab;* draw *ao*, cutting *xd* in *d;* draw *bo;* draw *dc*, cutting *bo* in *e;* and *abcd* is the inverted base of the pyramid.

To find its Centre by Vanishing Points.

The vanishing points *r*, *h*, for the diagonals of the plan, will also be the vanishing points for the diagonals in the inverted base; because all squares parallel to *mn*CD, and similarly opposed to the picture, will have the same vanishing points for their diagonals.

Therefore draw *br* and *ah*, crossing in *o*, the centre: draw *oe'* the axis of the pyramid; draw *e'a*, *e'b*, *ec'*, and *e'd*, and *fig.* 48 is perspectively represented on the right, in *fig.* 49.

To put a Wall in Perspective having a Passage formed in it.

Draw ME, *Plate* 11, *fig.* 50, equal 2 feet; draw MA perpendicular to ME, and equal 12 feet; draw ED parallel and equal to MA; join AD.

Draw EF in an oblique direction toward the right, and equal 24 feet; draw MG, DC, and AB, all equal and parallel to EF; join GF, GB, BC, and CF.

Suppose the entrance to be 12 feet wide, and 8 feet high.

Find X, the half of EF, lay off 6 feet from X to K, and from X to I, draw K*c* and I*f* parallel to ED or FC.

Make E*a* equal 8 feet; draw *af* parallel to

D C, and cutting K *c* and I *f* in *c* and *f*; draw K L, *c d*, I H, and *f e*; draw *a b*, in the end of the wall, parallel to A D; draw *b e*, cutting the last drawn lines in *d* and *e*; draw L *d* H *e* parallel to any of the vertical lines, and the wall is constructed.

The right hand side of the wall, *fig.* 50, we may call a vertical plane, as E D C F, with the entrance K *c f* I determined.

Required the Perspective of such a Plane.

On the right side of the picture, as at I, *fig.* 51, draw I C, produced upwards to *g*, making I *g* equal 12 feet; make I C equal 24 feet, and A B, the entrance, 12 feet.

Make I 2, in the vertical line I *g*, equal 8 feet; draw I *o*, 2 *o*, and *g o*; to *e*, the eye, draw lines from A, B, C, and they cut I *o* in *a*, *b*, and *c*; draw *a s* and *b r*, meeting the line 2 *o* in *r* and in *s*; draw *c h*; and the plane, with the entrance *a s r b*, as required, is represented.

On the left hand side of the picture draw the plan of *fig.* 50 as in *fig.* 51; draw also its seat G F *c b* equal M E D A, in *fig.* 50.

From G F *c b*, the seat of the end of the wall on the picture, draw lines to *o*, the vanishing point, and this will be the perspective of a wall infinite in length.

Make F *x* equal 8 feet, and draw *x o*; from I,

K, and G, in the plan to *e*, the eye, draw first I*e*, cutting F*o* in *i*; draw K*e*, cutting F*o* in *k*; and, finally, draw G*e*, cutting F*o* in *g*.

From *i* and *k* draw lines parallel to F*c*, cutting *xo* in *f* and *c*: and from *g*, cutting *co* in *d*.

Draw *xy* parallel to *bc*, and draw *yo*; draw *ih* and *kl* parallel to G′F, cutting G′*o* in *h*, and in *l*; draw *fe′* and *cd′* parallel to *xy*, cutting *yo* in *e′* and *d′*; join *he′* and *ld′*, which completes the perspective required.

If I stand at the end of a squarelike room, having a flat ceiling, and look toward the opposite ends, the ceiling and the side walls will seem to grow lower: *fig.* 51 may exemplify this appearance by joining *cg* and *hd*.

Here F*cdg* on the left, and I*cgh* on the right, will be the perspective of the sides of the room, and *cgdh* the ceiling, and lastly, *gdhc* be the opposite end of the room.

To construct a Step.

Draw AB, *Plate* 12, *fig.* 52, equal 5 feet; make A*e* and B*f* each perpendicular to AB, and equal one foot; draw BC in an oblique direction toward the right, and equal 5 feet; draw AD equal and parallel to BC; join DC.

Draw *eb* and *fg*, each parallel to BC, and equal 1 foot and a half, that is, one tenth, and half another tenth; join *bg*, raise *bi* and *gk*,

each perpendicular *eb*, or parallel to A*e*, and equal 1 foot; join *ik*: draw *cl* and D*m* parallel to B*f*, and equal 2 feet; join *im*, *ml*, and *lk*, as in the figure.

The foregoing in Perspective.

Imagine *fig.* 52 turned round to the picture, as on the left of *fig.* 54, by which the plan may be drawn.

The seat of the whole would cause confusion; the seat of the front step will suffice in this example.

Let AB*fé* be the seat of the front step; draw A*o*.

From *y*, in the plan, draw *ye*, cutting A*o* in a point too minute to be marked, but may be seen; from D, in the plan, draw D*e*, cutting A*o* in *d*: now draw *dm* parallel to A*é*; from that point where *ye* cuts A*o* draw a line parallel to *dm*; from *é* draw *eo*, cutting *hi* in *h*; draw *hg* parallel to *fe*.

Here B*x*, perpendicular to BA, is a line of measures.

Lay off 1 foot from *f* to *x*: draw *gk* parallel to B*x*; draw *xo*, cutting *gk* in *k*; draw *ki* parallel to *fé*; draw *io*, cutting *dm* in *m*; draw *ml* parallel to *fé*; and the perspective of *fig.* 52 is completed.

To construct a Cross.

Draw $f'e'$, *fig.* 53, equal 2 feet; draw $e'x$ and $f'w$ perpendicular to $f'e'$, and equal 11 feet; join xw; draw $f'c'$ in an oblique direction toward the left, and equal 7 feet and a half, that is, 7-tenths, and half a tenth; draw $c'u$ parallel and equal to $f'w$; join uw; then $c'uwf$ represent a quadrangular plane; draw $c'd'$ and uv parallel and equal $f'e'$; join $e'd'$, xv, and vd, and the figure represents a wall, and out of this figure may a cross be formed.

Let now the height of the middle bar of the cross be 11 feet, and 2 feet square at each end.

Find 3, the half $f'c'$, and lay off 1 foot from 3 to y, and from 3 to b'; draw yg and $b'd$; and $yb'dg$ is the plane of the vertical bar of the cross.

Draw yz, $b'a'$, gf, and de; draw zf and $a'e'$, and the upright bar is constructed.

Let the horizontal bar be the length of w, u, 2 feet square at each end, and 2 feet distant from the top of the vertical bar.

Lay off 2 feet from w to l, and from l to m; draw la and mr; and $ralm$ is the plane of the

cross bar; draw *lk*, *mn*, *hi*, *op*, *c5*, *bs*, *rs*, and *ab*; draw *ns* and *kb*, and the construction of the cross is evident.

To put Fig. 53 in Perspective.

Draw the plan as on the right hand side of the picture. *See Fig. 54.*

Draw *c'u* and *d'v* perpendicular to B*d'*, and they will be parallel to each other; make *c'u* and *dv* each equal 11 feet, and join *uv*; draw *co*, *d'o*, *uo*, and *vo*.

To *e*, the eye, draw *fe*, cutting *co* in *f'*; draw *f'w*, cutting *uo* in *w*, and *c'f'wu* is the plane of a wall in perspective.

Draw *f'e'* and *wf*, cutting *vo* in *f*; draw *e'f*, and the wall is represented.

To *e*, the eye, draw *b'e*, and it cuts *co* in *b*; draw *ye*, and it cuts *co* in *y'*; draw *bd* and *yb'* parallel to *c'u*, to meet *uo*, and *bdb'y* is the plane of the middle bar.

Draw *ba'*, *yz*, *de'*, and *bf'*, and draw *a'e*, *zf'*, and the middle bar is in perspective.

Lay off 2 feet from *u* to *a*, and from *a* to *t*; draw *ab* and *ts* parallel to *uv*; draw *ao*, cutting *f'w* in *l*; draw *to*, cutting *f'w* in *m*, and *almt* is the plane of the cross bar.

Draw *bo* and *so*; draw lines from each corner of the remote end of the cross bar, and

from the points where the plane of the cross bar, or horizontal bar cuts the plane of the vertical bar, all parallel to *uv*, and they will cross *bo* and *so* in the back plane of the cross bar.

The intersections are too minute to be noted by letters.

QUESTIONS, ETC. FOR PRACTICE.

Required the constructions of the figures in Plates 8, 9, 10, 11, and 12, on a scale to two-tenths of an inch to a foot.

If a line be drawn from the eye of the spectator parallel to an original line which is perpendicular to the picture, and in the plane of the horizon, where does the line so drawn from the eye cut the horizontal vanishing line?

What is that line called which is drawn from the seat of an original line to the vanishing point of that original line?

When an original line perpendicular to the picture coincides with the prime vertical, that is, with a line drawn from the eye through the centre of the picture, and through the original line, how is such an original line put in perspective?

When we obtain the indefinite representa-

tion of a line, to what point must lines be drawn from the original line to cut the indefinite representation, and represent the original line in perspective?

Has a line parallel to the picture any vanishing point?

How are the seat and perspective representation of such a line obtained?

How is the altitude of a pyramid found?

What kind of planes constitute a pyramid?

If two diagonals of a square be opposed to the intersecting line in the horizon, and the angle of 90^0 opposite the picture, what kind of an angle, at the eye, will two lines drawn from the eye parallel to those diagonals make?

If we bring down or transfer the eye to the horizontal line, to what line or plane do we transfer the original line?

In the following theorems we mean all right or straight lines.

If a line be drawn through the eye parallel to the horizontal line, this line is called the parallel of the eye; the parallel of the eye is also parallel to the intersecting line of the picture, or ground line.

If a line be drawn cutting the intersecting line, or ground line, and parallel of the eye, this line will make the angles, with the inter-

secting line and parallel of the eye, respectively equal.

If a line be drawn from the eye parallel to an original line in the plane of the horizon, the line so drawn from the eye will make an angle with the parallel of the eye, equal to the angle which the original line makes with the intersecting or ground line.

The angle, which a line drawn from the eye parallel to an original line makes, with the prime vertical, is always equal to the complement of the angle, which the original line makes with the intersecting line, because the prime vertical coincides with the distance of the eye from the picture. For, on the same side, the angle made by the parallel of the eye, and the line parallel to the original line, is equal to the angle which the original line makes with the intersecting line; hence, one must be the complement of the other, for the parallel of the eye and prime vertical are at right angles, in the subsequent kind of constructions. Thus, suppose an original line to make an angle of 40^0 with the intersecting line, a line drawn from the eye, parallel to this original line, will make an angle, with the parallel of the eye, equal 40^0; and the same parallel of the original line will make an angle,

with the prime vertical, equal 50^0, the complement of 40^0; hence, if we use the complement of the angle which the original line makes with the intersecting line, we can dispense with the parallel of the eye.

Of Planes perpendicular to the Picture, and to the Horizon.

To find the vanishing line of such planes.

If a plane be imagined to pass through the eye parallel to an original vertical plane, perpendicular to the picture, and to the horizon, it will cut the picture in the prime vertical, that is, it will be a vertical vanishing line passing through the centre of the picture.

If a line be drawn from the eye parallel to any original line in this vertical plane, it will cut the vertical vanishing line in the vanishing point for the original line, in the vertical plane.

Of a Plane inclined to the Horizon, and intersecting a Vertical Plane.

Let the roof of a plain house, whose ridge, or top edge of the roof, is parallel to the picture, be given to exemplify this theory.

The prime vertical will be the vanishing line for the two sides of the house, perpendicular to the picture.

If a plane pass through the eye parallel to

the near half or side of the roof, it will cut the picture above the horizontal vanishing line, and produce the vanishing line for the near side of the roof; and a plane passing through the eye, parallel to the other or remote side of the roof, will cut the picture below the horizontal line, and produce its vanishing line.

If a vertical plane intersect an inclined plane, the line of their intersection will be alternately in each plane, that is, in the vertical plane, and in the inclined plane.

To find the vanishing Point for the Edges of the gable End, that is, for the Intersection of the gable End with the Roof.

Let the inclination of the roof to the horizon be in an angle of 30°, declining from the picture.

Now this edge, or end of the roof, is in a vertical plane, as well as in the roof; and the vertical side of the house perpendicular to the picture, has the prime vertical for its vanishing line.

Let, therefore, a line be drawn from the eye, making an angle of 30° with the horizon; and this line determines the vanishing point for the edge of the roof above the horizontal line, in the vertical vanishing line. A line drawn from the eye, making the same angle with the hori-

zon downward, determines the vanishing point for the off side of the roof, in the vertical vanishing line, below the horizontal vanishing line.

To put a Roof in Perspective detached from the House, and lying on the Horizon, the Plane of the gable End being perpendicular to the Picture; then the Ridge of the Roof is parallel to the Picture.

Draw B*l*, *Plate* 13, *fig.* 55, indefinitely; draw BG inclined toward the right, and equal 15 feet; draw G*k* indefinitely, and parallel to B*l*.

Make BA equal 14 feet; draw A*v* indefinitely, and perpendicular to B*l*; draw AH parallel to BG; find 3, the half of AH, and draw 3K and H*w*, both parallel to A*v*.

Make 3*o* equal 5 feet, and draw 3*f* and *oe'* parallel to A*l*, each equal 15 feet, and join *e'f'*.

Here we shall be obliged to use irregular proportions of a house, in order that the perspective may appear more pleasing to the eye.

Let BA, equal 14 feet, be the length of the gable end; find 2, the half of BA, and raise 2E perpendicular to BA, and equal one foot and a half; join EB and EA.

Make BC and AD equal 5 feet; join CD;

draw 2, 4 parallel and equal to BC; draw 4F parallel and equal E2; join EF; join FC and FD.

Here BCDA is the base of the roof, and we may find its perspective thus:

AD is the seat of the base on the picture, and *o* its vanishing point; therefore draw A*o* and D*o*; from B and C to *e'*, the eye, draw B*e'*, cutting A*o* in *b*; from C draw *ce'*, cutting D*o* in *c*; join *bc*, and A*bc*D is the perspective of BCDA, the base of the roof.

AEFD is the near side of the roof, and inclined to the horizon in an angle of 11° 30′.

To find the vanishing line for this plane, we proceed thus:

With the chord of 60° and centre *e'*, describe the arc P*mn*; make *mn* equal 11° 30′, and as CFEB, the remote side of the roof, makes 11° 30′, with the horizon also, lay off 11° 30′ from *m* to *p* downward; draw from *e'*, through *n* and *y*, lines produced to cut KL in *x* and *y*.

Now draw, or imagine to be drawn, a line through *x* parallel to *hor*, and it will be the vanishing line of the plane EFDA of the roof; again, imagine a line to be drawn through *y* parallel to *hor*, and it will be the vanishing line for the plane BCEF, the remote side of the roof.

Here, as the lines AE and DF are in two

parallel vertical planes perpendicular to the picture, they will vanish in x; therefore, draw Ax and Dx.

Again, the lines BE and CF are in the same vertical planes as the former; therefore, draw yb, produced back to cut Ax in e; draw yc, produced back to cut Dx in f; join ef; and Ae, bc, cf, Dc is the perspective of the original roof.

By the general Method.

Draw rA, *fig.* 56, the intersecting or ground line; draw $z\acute{e}$, the horizontal line, 5 feet distant from rA, the intersecting line; lay off the distance of the eye from o to e'.

Draw ABCD, the plan of the roof on the right, the same dimensions as in *fig.* 55.

DA is the seat of the plan on the picture, and o is the vanishing point.

Draw Do and Ao; draw Ce', cutting Do in c; draw cb parallel to DA, and the plan is in perspective.

Bring down e', the eye, on ze' the horizontal line, by making $o\acute{e}$ equal oe'.

The inclination of the roof is the same as before, viz. $11^0\ 30'$; with the chord of 60^0, and centre $\acute{e}$, describe the arc mn; make mn equal $11^0\ 30'$, and through n draw $\acute{e}n$, produced to cut KL in x; make oy equal ox, and y is the

vanishing point for the remote side of the roof.

A line drawn through x parallel to ze'' is the vanishing line for the near side of the roof; and a line drawn through y, parallel to ze', would be the vanishing line for the remote side of the roof.

Now x is the vanishing point for the near side of the roof; therefore, draw Dx and Ax.

y is the vanishing point for the other side of the roof: draw yc, produced back to cut Dx in f; draw yb, produced back to cut Ax in e; draw fe, and the roof is in perspective.

The vanishing points, x and y, for the sides of the roof, will be the same, whether the original roof be above or below the horizontal line.

To put a House in Perspective.

Let $rPqt$, *fig.* 56, on the left, be the plan of a house, having the same dimensions as the plan of the roof on the right.

Draw Pk and ri perpendicular to rA, and equal 6 feet; join ik; and $rikp$ is the seat of the end walls of the house on the picture.

o is the vanishing point for the front and its parallel, &c.

Draw Po, ro, ko, and io; to the eye, draw

qe', cutting po in s; draw sl parallel to pk, and complete the other lines, as dotted in the figure.

For the Roof.

The inclination of the sides of the roof the same as before; hence x and y are the vanishing points.

Draw kx and ix; draw yl, produced back to cut kx in w; draw yz, produced back to cut ix in v; draw vw, and the house is in perspective as proposed.

The student, for practice, is requested to construct *figs.* 57 and 58, the dimensions and inclination of the roof being the same as in *fig.* 56, *Plate* 13.

To put a Street in Perspective.

Let the plan on the right, *fig.* 59, *Plate* 14, and the plan on the left, be the same dimensions as before.

Find the seats of the end walls on the picture; x and y the vanishing points for the roof as before: here also the pupil may put the first without the roof in perspective, as on the left in *fig.* 56, *plate* 13.

In the new perspective, if we have the vanishing points for the roof of one house, we

may put any number of houses, having the same dimensions, and similarly opposed to the picture, in perspective.

Thus, in *fig.* 59, *x*, in the vanishing line 5, 6, is the vanishing point for the near side of the roof, and *y*, below the horizontal line, the vanishing point for the remote side of the roof.

Draw *dx* and *e'x*; draw *yb*, cutting *do* in *g*; draw *gf* parallel to *hr*, which is the ridge of the roof.

Here it is plain that the gable end of any number of houses will appear in the triangle *dgo*.

Now to draw the second house without a plan, we draw from *d*, its supposed corner, the line *dx*, cutting *go* as in the figure; then, from this point in *go*, which is the vertex of the required gable end, draw a line to *y*, and where this last drawn line cuts *do*, determines the breadth of the required house; from which draw a line parallel to *dc*, to cut *co* on the ground; draw also from the vertex or top of this second house, a line parallel to *de'*, and it will be the visible part of the ridge, as in the figure; and thus, for any number of houses.

The two houses on the right may be represented, as in the figure, by the same method; but the pupil constructing this figure will please to leave greater space for the street.

Sometimes the following method is used to put the roof of a house in perspective.

Let *cdb* be the front of a house; draw two diagonals, as from *b* to *d*, and from the other opposite corners, but they are here omitted; then, from the point of their intersection, draw a line parallel to one corner or side, as *dc*, upward.

Let *cd* produced, be a line of measures for verticals, lines, or planes; lay the height of the gable end from *d* to *b'*, and draw *b'o*, cutting the vertical line drawn from the intersection of the diagonals, and it cuts this line as supposed, in *g*; join *gd* and *ga*, and the gable end is determined; draw *gf* indefinitely; then, by a process similar to that used in the front, find *e'f*, or, by drawing a line from the perspective middle of *da*, and produced to cut *e'o* (here invisible); draw, from this point, a line parallel to *ga*, cutting *gf* in *f*, and join *e'f*.

This is an old tedious method, and not general for more than one house.

Before we proceed to the construction of the figures in Plates 15 and 16, it may be necessary to premise a few particulars relative to the transposing of the eye from its original place, to other places in various vanishing lines.

To explain this, let us imagine a plane having a circular boundary, like the upper part or

plane of a round table: let us imagine the diameter of this circular plane 10 feet, and elevated 5 feet *above*, and parallel *to*, the horizon.

Now let us imagine a plane, as the plane of the picture (spoken of in the preceding operations), to pass through the centre of this circular plane, and assume a vertical position; then the plane of the picture will be at right angles to the circular plane, and their intersection may be called the horizontal vanishing line.

Let us suppose the eye of a spectator to coincide with the edge of the circular plane, in a point from which a line being drawn may pass through the centre of the circular plane, and be at right angles to the horizontal vanishing line; and, consequently, at right angles to the plane of the picture; hence the centre of the circular plane will be the centre of the picture.

Let us imagine an original straight line beyond the picture; and, perpendicular to the picture, its vanishing point will be the centre of the picture.

From the seat of this original line in the picture, draw a line to the centre of the picture, its vanishing point; and this will be the indefinite representation of the original line.

From each extremity of the original line,

draw a line to the eye, cutting the indefinite representation, and the portion of the indefinite representation intercepted between the lines drawn to the eye, will be the perspective of the original line.

Here it is evident that the eye, the original line, and its perspective representation, are all in one plane; which is of a triangular form, made by the two lines drawn to the eye, and by the original line; this last line we may call the base of the triangle.

If we suppose the eye moved to any other point in the edge of the circular plane, and the original line so transposed that it, and its indefinite representation, may be in the same plane, that is, in one plane, the perspective of any original line perpendicular to the picture, may be determined; and indeed that of others.

To exemplify this theory, see *Plate* 7, *fig.* 39, where *e* is the eye, I *uv* T the picture, *o* the centre of the picture, *hor* the horizontal vanishing line, L I, in the line A B, an original line perpendicular to the picture, and I *o* the indefinite representation of L I.

From L to *e*, the eye, draw L *e*, cutting I *o*, the indefinite representation in the point *l*; imagine a line drawn to *e*, the eye, from I, the near extremity of L I; then will *e* L I *e* form a

triangular plane, in which are *e*, the eye, 1 *l*, and LI, the original line.

Here the plane in which *e*, the eye, is placed, is not circular, but quadrangular; and if it were circular, no more than half would appear: however *o*, the centre of the picture may here be considered as the centre of the circular plane.

Let it be proposed to find the perspective of LI, by transposing *e*, the eye, to *h o r*, the horizontal line.

With the radius *oe*, and centre *o*, describe an arc from *e* to *r;* and *r* is the transposed place of the eye.

In this example LI the original line is not in the same plane as *r*, with the indefinite representation 1*o;* wherefore produce TI to G; then with the radius IL, and centre I, describe an arc from L to G; and GI, equal IL, is transposed to the plane in which *r*, the eye, and 1*o*, the indefinite representation, are placed.

Now draw from G to *r*, the eye, the line G*r*, cutting 1*o*, the indefinite representation, in the point *l;* imagine a line drawn from I to *r*, and the triangle *r* G I *r* is the triangular plane, in which is seen 1 *l*, the perspective of GI, which is equal to LI, the original line: hence this operation agrees precisely with the preceding, when the eye was placed at *c*.

If *e*, the eye, be transposed to *h*, in the horizontal line *h o r*, and a process used similar to the last, the result will be the same as by the last.

But in *fig.* 40 the process may seem different to the learner; yet any doubt respecting this is easily obviated.

In *fig.* 40 the picture and the spectator are made to coincide with the plane of the horizon; as also, the half of the imaginary circular plane.

But the line *o e*, in *fig.* 40, corresponds to *o s* in *fig.* 39, and *o s* is equal *o e*.

Let *u* I, in *fig.* 39, be produced to *g*, and I *g* made equal to I L; then suppose the eye at *s*, I *o*, the indefinite representation, and I *g*, the original line, are all in one plane.

Draw *g s*, cutting I *o* in *l*; imagine a line drawn from I to *s*, and the triangular plane is again obtained, in which is found I *l*, the perspective of I *g*, equal I L, as in the two preceding operations.

The correspondence of *fig.* 39 to *fig.* 40 may be proved thus: imagine the picture in *fig.* 39, and *f e*, the spectator, to fold over on the side of the spectator; then *u g* will turn over on I T; I *g* will coincide with I L, *f'* on the picture with *f* on the ground, the feet of the spectator,

and *s* above the picture with *c*, the eye of the spectator.

By attentively comparing the letters and lines in *fig.* 39 with those in *fig.* 40, the learner will soon perceive the similarity of the preceding operations with those in *fig.* 40.

Transposing the eye is, on many occasions, found to be of great importance; and this transposition may consist of two varieties.

First, the eye may be brought down, as it is sometimes called, to cut off from the indefinite representation of a line, a portion equal to the original of that portion: and, secondly, the eye may be transposed from its original place for the horizontal vanishing line, to an appropriate place for any other vanishing line.

We will first speak of bringing down the eye as in the first variety, and endeavour to explain its utility.

Every line not parallel to the picture has a vanishing point on the picture; and if a line be drawn from such a vanishing point to the eye, this line is parallel to the original line; and shows the distance of the eye from such a vanishing point.

If one foot of the compasses be placed in this vanishing point, and the other extended to the eye, then with this radius, and the va-

nishing point as a centre, transpose the eye to the vanishing line, in which the vanishing point or foot of the compasses is placed; and a line or lines drawn from the transposed place of the eye to cut the indefinite representation of a line; and this line, or these lines, drawn from the eye, produced to cut the intersecting line of the plane in which the original line is placed, the portion cut off from such an intersecting line will be equal to the original of its perspective representation.

To exemplify this theory, let 1 *l*, *fig.* 40, be the perspective representation of an original line, to find its original.

Produce 1 *l* to *o*; draw a line from *e*, the eye, to the vanishing point *o*.

T 1 is the intersection of the original plane, in which the original of 1 *l* is placed.

Place one foot of the compasses in *o*, the vanishing point of 1 *l*; extend the other to *e*, and transfer *e*, the eye, to *r*, in the horizontal line; draw *r l*, produced to cut the intersecting line T 1 produced in G; and 1 G is equal the original of 1 *l*.

Or *e* transferred to *h*, and *h l* drawn and produced to cut T 1 in 3, gives 3 1 equal the original of 1 *l*, as before.

The utility of bringing down the eye, in order to put an original line in perspective,

may be seen in *fig.* 41, *Plate* 8, where the original line $k\,l$ is, in the prime vertical $e\,i$, produced: but bringing down e, the eye, to h or to r, and transposing the original line $k\,l$ to P z, the intersecting line for h: or to the right, for the eye at r, its perspective may be obtained, as may be seen in the figure.

The converse of the above operation is also easily performed thus:—In *fig.* 41, *Plate* 8, let $x\,y$ be the perspective of some original line in a horizontal plane, of which $p\,z$ is the intersection; produce $x\,y$ both ways, or, on the present occasion, only to o, its vanishing point: with the radius $o\,e$, and centre o, transfer e to h.

Through x and y draw two lines, as first, $h\,x$, produced to cut the intersecting line $p\,z$ in p; and draw $h\,y$, produced to cut $p\,z$ in r', and $p\,r'$ is the original of $x\,y$; as in the first example.

Again, let A g, *fig.* 63, *Plate* 16, be a line given in perspective to find its original, which is in an horizontal plane; W V is the intersection of the original plane in which the original line is placed, and $b\,i$ is the vanishing line of the original plane.

Produce A g to i; join $i\,e$, and $i\,e$ is parallel to the original of A g; from A draw A D; then from e, through g, draw a line (which is here

omitted), produced to cut A D in D, and A D is the original of A *g*. Here A D is found in its true place.

Or, by bringing down the eye thus, with the radius *i e*, and centre *i*, bring down the eye at *e* to T, on the horizontal vanishing line; then draw, through *g*, the line T *g*, produced to cut the intersecting line W V in *z*, and A *z* is the original of A *g*, equal A D, found by the former or first method.

To put A *z* equal A D in perspective; *i* is its vanishing point.

Here the original line A D cuts the intersecting line obliquely; wherefore, draw A *i*: from *i*, with the radius *i e*, transpose *e*, the eye, to $\overset{T}{x}$; from $\overset{T}{x}$ to *z* draw $\overset{T}{x}z$, cutting A *i* in *g*, and A *g* is the perspective of A *z*, or of its equal A D.

Hence lines, &c. may be put in perspective by having the intersection of the plane in which such lines are placed; as also the intersecting point or points of an original line or lines, with such an intersecting line.

Before we give any more examples of bringing down the eye, it may be proper to speak of vanishing lines, and of their intersections.

As original planes may have various degrees of inclination to each other, determined by their intersections, so their perspective repre-

sentations will vary the inclinations of their vanishing lines.

In a common house, consisting of four walls, there are two respectively parallel to each other; that is, the two sides are parallel to each other; and also, the two ends are parallel to each other.

If we have the vanishing line and vanishing point of one wall or plane, the same vanishing point will answer for all walls, planes, and lines parallel to the first wall, plane, or line.

But the roof of a house inclining to the horizon will have a different vanishing point.

Thus, suppose two walls or two planes perpendicular to each other.

If one of these planes be perpendicular to the picture and to the horizon, the other plane or wall will be parallel to the picture.

Then the centre of the picture will be the vanishing point for the wall or plane perpendicular to the picture; the other wall or plane will have no vanishing point, being parallel to the picture.

Let there be the half or one side of a roof so placed on the above vertical planes, that the lower edge of the side of the half roof may coincide with the top of that plane, which is parallel to the picture, and one end of the roof

be in that plane produced upward, which is perpendicular to the picture; and let the inclination of the half roof be any number of degrees to the horizon; and let it decline from the picture.

Now the wall or plane which is perpendicular to the picture, and in which is the end of the roof, will have the same vanishing line; but the vanishing point for the wall will be in the horizontal vanishing line; because the top and bottom of the wall are horizontal lines.

Through the vanishing point of the wall, which is the centre of the picture, draw an indefinite right line, perpendicular to the horizontal vanishing line, and this will be a vertical vanishing line for all original lines in that wall, or in planes parallel to that wall.

Now bring down the eye to the horizontal line, and with the chord of 60^0, and centre, the transposed place of the eye, describe an indefinite arc through the horizontal line: if the plane of the roof decline from the picture, lay off the chord of the angle of inclination on the arc from the horizontal line upward, and mark this point.

Through this point and the transposed place of the eye, draw a line produced to cut the vertical vanishing line, above the horizontal vanishing line: this is the vanishing point for

both ends of the half roof, when it declines from the picture; but the chord of the inclination must be laid off downward, when the roof inclines to the picture.

Here the horizontal and vertical vanishing lines are connected, because the wall, perpendicular to the picture, is connected with the end of the roof; the former vanishing in the horizontal vanishing line, and the latter vanishing in a vertical vanishing line.

The learner is requested to consider the figures in *Plates* 13 and 14, and to read attentively the process of operation for each figure.

Of the Centre and Distance of a Vanishing Line.

Every vanishing line has a centre, and this centre is determined by letting fall a perpendicular, from the centre of the picture to such a vanishing line: this perpendicular produced is the vertical for that vanishing line, and on this vertical, from the centre of such a vanishing line, the appropriate distance of the eye is laid off.

The usual method of transposing the eye from its original place for such vanishing lines, is thus:

Through the centre of the picture draw a line parallel to the given vanishing line; lay

off the original distance of the eye from the centre of the picture toward the right or left of the centre, on this last drawn line, and mark that point or limit: from this point draw a line to the centre of the given vanishing line, and this is the distance of the vanishing line from the original place of the eye.

In *fig.* 59, *Plate* 14, it is evident 5, 6 is the vanishing line for the roofs of the houses.

To find the Centre of 5, 6.

From *o*, the centre of the picture, draw *o x*, perpendicular to 5, 6, and produce *o x* to *e'*.

To lay off the Distance of the Eye from x to e'.

Through *o*, the centre of the picture, if not already drawn, draw *h o r* parallel to 5, 6.

Make *o h* on the left, or *o r* on the right, equal *o e*, the original distance of the eye; and draw, suppose, *h x*.

Here is a right angled triangle, the base of which is *o h*, equal the original distance of the eye; the perpendicular *o x*, the distance of the vanishing line from the centre of the picture; and the hypothenuse *h x*, the distance of the vanishing line from the original place of the eye.

To make this plainer to the pupil, let him draw such a figure as 59; then cut lightly not

quite through the vanishing line 5, 6, and turn it over; let him also cut $o\,h$ and $x\,h$, quite through, and cut $o\,x$ lightly; then turn round the triangle $o\,h\,x$, until it become perpendicular to the plane of the paper; turn down $x\,e'$, and e' and h will coincide in the original place of the eye.

Pierce two holes through the intersecting line produced through $m\,c$, and on the back, through these holes, draw a line cutting the paper lightly; then turn up the ground plane, where the plans of the houses are; then that part above 5, 6, while e' is at the original place of the eye, will be the plane passing from the eye parallel to the visible sides of the roofs, and producing the vanishing line 5, 6 on the picture.

Supposing *fig.* 59 to remain as before, we have two vanishing lines, and they are parallel to each other; namely, $h\,o\,r$, which must always appear on the paper, and 5, 6, which is the vanishing line for the inclination of the near side of the roof $c'\,f\,b\,d$, and for all planes parallel to $c'\,f\,b\,d$: and if a line be drawn through y, parallel to $h\,o\,r$, it will be the vanishing line for the other half of the roof, and for all planes parallel to that half.

Because $f\,b$, the ridge of the roof $c'\,f\,b\,d$, and all the other ridges, are parallel to the picture,

they will have vanishing lines parallel to *h o r;* but if the ridges were inclined to the picture, they would have inclined vanishing lines, as in *figs.* 61 and 63, *Plates* 15 and 16.

Now, as the fronts of the houses and the ends of the roof vanish in *e x y*, the same vertical vanishing line, it will be necessary to transpose the eye for *e x y;* which is done by taking *o e* in the compasses as radius, and centre *o*, transferring *e* round to *r*, or to *h;* if to *h*, then *h* is the place of the eye for *e x y*, and cuts off from the intersecting line *m c* produced, or from any intersecting line parallel to *m c q*, portions equal to the perspective lines vanishing in *o*, the centre of the picture, and centre also of *e x y*.

Lines of Measures, &c.

The plane of the picture is the standard for determining the dimensions of original objects: so that any original plane, cutting the picture, produces a section on the picture, and this section becomes a line of measures for ascertaining the original dimensions of original objects, on the plane by which the section is made.

If a line or plane coincide with the picture, that line or plane preserves its original dimen-

sions: that is, such line or plane is not altered by any perspective operation.

In *fig.* 59, *Plate* 14, *m c q*, produced both ways, is the intersection of an original horizontal plane with the plane of the picture; hence this intersection becomes an horizontal line of measures for all original lines in this horizontal plane.

On the left, in *fig.* 59, *m c* is the bottom or lower edge of the near end of the house, and here *m c* coinciding with the intersecting line, or line of measures on the picture, cannot admit of any change by perspective.

Again, the near end of the house is *m e' d c*, and this end or plane coincides with the picture, and cannot, by perspective, be changed from its present dimensions.

But A B, the lower edge of the remote end, and the end plane of the same house being removed from the picture, are altered by perspective, in all their dimensions, as may be concluded from the figure.

Here *h o r*, the horizontal vanishing line, is parallel to *m c q*, the intersecting line.

5 *x* 6 is a vanishing line parallel to *m c q;* but 5 *x* 6 was not obtained by a plane passing from *e*, the eye, parallel to an original horizontal plane; but by a plane passing from *e*, the

eye, parallel to that half of the roof which declines from the picture; and the intersection of the lower side of this roof is *e' d k:* hence *e' d k* is a line of measures for determining the fronts of all such roofs, according to their original dimensions.

Finally, *exy* is a vertical vanishing line, and *cz* and *wv* are the lines of measures for the fronts of the houses, because the original planes in the fronts of the houses, produced upward, cut the picture in *cz* and *wv*, and produce these vertical lines of measures.

We now request the pupil's attention while we endeavour to show the methods of finding the originals of lines in perspective, which is called the inverse method of perspective.

Let an horizontal vanishing line, the place of the eye, an horizontal line of measures, and the perspective of some original line, in that plane whose intersection with the picture makes the above line of measures, be given, to find the original of the line in perspective.

In *fig.* 59, *Plate* 14, let *hor* be the horizontal vanishing line, *h* the transposed place of the eye, *mcq* the line of measures, and *cb* a line in perspective, vanishing in *o*, it is required to find the original of *cb*.

First, *c*, in *mcq*, coincides with the picture;

through *h b* draw *h b*, produced to cut the line of measures in *q;* and *c q* is the original of *c b:* this *c q* is equal *c* B in the plan.

But the line *c b* is also in the vertical plane *c d a b;* hence *c b* may be found by a vertical line of measures.

e x p is a vertical vanishing line, and *r*, the eye; but suppose it transposed to *p*, in the vertical vanishing line.

c d z is a vertical line of measures made by the fronts of the houses on the left, produced upward.

c, as before, remains unchanged, therefore from *p*, through *b*, draw *p b*, produced to cut *c z* in *z;* and *c z*, equal *c q*, equal *c* B, is the original of *c b* as before.

The converse or direct process of the preceding examples may be easily performed.

As before, *m c q* is the line of measures, *h o r* the horizontal vanishing line, *h* the transposed place of the eye, and *c o*, the indefinite representation of a line, vanishing in the centre of the picture.

Let *c q* be equal the original.

From *q* to *h* draw *h q*, cutting *c o* in *b;* and *c b* is the perspective of *c q*.

Again, *e x p* is a vertical vanishing line, *r* the eye, but transposed in *p; c z*, the original line

in a vertical line of measures, *c o* the indefinite representation of *c z*.

Draw *z p*, cutting *c o* in the point *b*; and *c b* is the perspective of *c z*, as before.

To find the Original of a vertical Line in Perspective.

This is easily performed.

In *fig.* 42, *Plate* 8, is B *d c a*, a vertical plane in perspective; the near end B *d* coincides with the picture, and remains unaltered; the remote end *a c* is shorter; *h o r* the horizontal vanishing line; B *d k* a line of measures; *c a* a line in perspective, in a plane which vanishes in *o*, the centre of the picture.

Through *a* draw *o a*, produced to B; draw *o e*, produced to *d*; and B *d* is the original of *c a*.

Or let *h* be the transposed place of the eye for *e o s*, *x y* the transposed line of measures; through *h* and *a* draw *h a*, produced to *x*; through *h* and *c* draw *h c y*; and *x y* is equal the original of *a c*.

Or, lastly, let *s* be the transposed place of the eye, in a vertical vanishing line, as *e o s*, passing through *o*, the vanishing point of the plane in which *c a* is placed.

Through *s* and *a* draw *s a*, produced to cut

the line of measures B *d* produced, in the point *k*.

Through *c* and *s* draw *s c*, produced to cut the line of measures, in the point *z*, in *fig.* 43; and *k z* is the original of *c a*, as before.

In *fig.* 59, *Plate* 14, let 5 *x* 6 be the vanishing line of the near side of the roof declining from the picture; *e' d k* the line of measures, *ë* is the place of the eye, but G is its transposed place, it is required to find the original of *t i*, the near edge of the roof, in the first house on the right.

The corner *t* coincides with the picture, and remains fixed: through G and *i* draw G *i*, produced to cut the line of measures *e' t k* in *k;* and *t k* is the original of *i t*, equal *b c* or *a c*, *fig.* 58.

Again, *t i* being in a vertical plane, as well as in an inclined plane, its original may be found by a vertical line of measures.

Let *e x p* be a vertical vanishing line, *ë* the eye which the radius *x ë* has transposed round to *ï*, in the lower part of the vertical vanishing line; *w v*, a vertical line of measures.

t, the corner, as before remarked, remains fixed.

From *ï*, through *i*, draw a line produced to cut the line of measures *w v*, in the point *v;* and *t v* is the original of *t i*, as before.

Having given some examples of transposing the eye for three different kinds of vanishing lines, in order to determine the originals of lines in perspective, we will now speak a few words of transposing the eye to vanishing lines made by planes inclined to the picture; because the foregoing examples consisted of planes (we mean the walls of the houses) parallel and perpendicular to the picture.

In the foregoing examples of houses, the ridges of the roofs were parallel to the plane of the picture.

In the following example the ridge will be inclined to the picture.

In *fig.* 63, *Plate* 16, W A V is the intersecting line made by an original horizontal plane; that is, by the plane of the ground; and hence by the old writers called the ground line; *e* is the original place of the eye.

A B C D is the plan of a house: and the next house to the intersecting line; W A V is the same house in perspective; *i*, in the horizontal line, is the vanishing point of the lower and upper edges of the front of the house; and the vertical vanishing line *k i l*, passing through *i*, is the vanishing line for the ends of the roof; because the ends of the roof are in the same plane as the front of the house; and as the

ends of the roof join the upper edge of the front wall, this edge is horizontal, vanishing in *i*; so the vertical vanishing line *k i l*, for the same reason, is joined *to* or passes *through i*, in the horizontal vanishing line.

b, on the left, in the horizontal line, is the vanishing point for the left end or side of the house; that is, for the lower and upper edges of the wall; and for B *s*, the ridge, which is also parallel to those edges.

Then because the ends of the roof vanish in a vertical vanishing line, that is, in *k above*, and in *l below*, the horizontal line, and the ends and the ridge vanish in a horizontal vanishing line; therefore, the vanishing point *k*, in the vertical vanishing line, and the vanishing point *b*, in a horizontal vanishing line, must be joined by the straight line *b k*, as in the figure: and the line *b k* is the vanishing line of the roof, that is, for the near half of the roof which declines from the picture.

In *fig.* 59, *Plate* 14, the front or fronts of the house or houses vanish in the centre of the picture: and the ridge of the roof or roofs is parallel to the picture.

The ridge of the roof in *fig.* 59 has no vanishing point, therefore, the plane imagined to pass from the eye parallel to the original, or

near side of the roof, cutting the picture, makes the vanishing line for the roof parallel to the horizontal vanishing line.

But in *fig.* 63, *Plate* 16, the plane passing from the eye parallel to the original near side of the roof, produces an inclined vanishing line for the near side of the roof; because the ridge is inclined to the plane of the picture.

It is true, the vanishing line *bk* for the roof in *fig.* 63, was not obtained by the foregoing imaginary process, but by joining the vanishing points *b* and *k*.

Having obtained the vanishing line *bk*, it is required to find its centre and distance from the original place of the eye.

From *o*, the centre of the picture, let fall a perpendicular on *bk*, as *oo'* produced toward *n*; and *o'* is the centre of *bk*.

For the distance, draw *o*w parallel to *bk*; make *o*w equal *oe*, and join *ó*w.

With the radius *o'*w, and centre *o'*, transfer the point w round to *n*; and *n* is the appropriate place of the eye, for the vanishing line *bo'k*; or with the radius *be*, and centre *b*, transfer *e*, the eye, round to *n*, which produces the same effect.

To find the Line of Measures for b o k.

The near corner of the house is A *m*, which coincides with the picture; and hence remains fixed.

The roof begins at *m;* now the lower edge of the original roof is supposed to cut the picture, and produce a section parallel to the vanishing line *b o k*.

Therefore, through *m*, the corner of the house, draw a line *m* 6, produced both ways, and parallel to *b o k;* and this is a line of measures for measuring all parts of the near side of the roof.

To bring down the eye for *b o k*, place one foot of the compasses in the point *k*, open the other to *n;* then with the radius *k n*, and centre *k*, transfer *n*, the eye, to *e;* and being thus prepared, we can measure any part of the near side of the roof.

b o i is the horizontal vanishing line, *e* the eye, W A V the line of measures.

With the radius *i e*, and centre *i*, bring the eye at *e*, its original place, to T, in the horizontal line; and thus we can measure all lines vanishing in *i*.

k i l is a vertical vanishing line, because *o i* is

perpendicular to *k i l*, and *i e* is its distance from the eye at *e*.

Imagine *b o i*, produced beyond the limits of the paper.

Then, with the radius *i e*, and centre *i*, transfer *e*, the eye, round toward the right, so as to cut *b o i* produced; and this will be the proper place for the eye.

But this method is often found inconvenient, as falling beyond the limits of the paper; wherefore, recourse is had to another method, which is called bringing down the eye on the inside; the former is bringing down the eye on the outside of the vanishing line *k i l*.

The latter method is thus performed: with the radius *i e*, and centre *i*, bring down the eye to T, in the horizontal line; and from T bring it round to *q*, in the vertical vanishing line.

Now as A *m*, the corner of the house, coincides with the picture, through A *m* draw a vertical line, as A *r*, produced upward; and this will be a vertical line of measures.

With the radius *k e'*, and centre *k*, transfer *e'*, the eye, down to 7, in the vertical vanishing line, and the figure is prepared to find the originals corresponding to any parts of the house.

To find the Originals of Lines that vanish in the horizontal vanishing Line b o i.

Let it be required to find the original of A *g*, the lower edge of the front of the house.

T is the transposed place of the eye, and W A V the line of measures: produce T *g* to *z*; and A *z* is the original of A *g*, equal A D in the plan, as mentioned in another place.

But A *g* is in a vertical plane, and hence it may be measured by means of a vertical vanishing line and a vertical line of measures, thus: *k i l* is the vertical vanishing line passing through *i*, the vanishing point of A *g*; A *r* the vertical line of measures, and *q* the transposed place of the eye.

From *q*, through *g*, draw a line produced to cut A *x*, in the point *x*, and A *x* is equal to A *z*, equal A D, as before.

The end of the house A *x* is measured in a similar manner; namely, by bringing down *e*, the eye, on the right, with the radius *b e*; but is here omitted, being left for the pupil's practice.

To find the Original of a vertical Line, suppose of the remote Edge of the sixth House, in Fig. 63.

i is the vanishing point of the fronts of the house, and A*r* a line of measures.

From the remote lower corner 5 draw *i* 5 A; from the upper corner below the roof, and directly over 5, draw a line from *i*, produced to *m*; and A *m* is the original of the remote edge of the sixth house.

The same may be found thus: *i q*, in the vertical vanishing line, is equal to *ie*, or to *i* T, in the horizontal vanishing line.

Suppose A*r* produced upward, then a line drawn from *q* through 5, and another from *q* through the corner over 5; these two lines produced to cut A*r* produced, would intercept a portion of A*r* produced, equal A *m*, as before.

To find the Original of a Line in the near Side of the Roof.

n is the eye, and *e'* its transposed place.

A line drawn through *m* parallel *b o' k*, as *m* 6, is the line of measures.

To measure *m* B, the near end of the roof: *m*, as before remarked, coincides with the pic-

ture, and remains fixed; therefore, one line drawn from e' will determine its original.

From e', through B, draw e' B, cutting $m6$ in a point a little to the right of B; and this extent is its original, equal bd or de, in *fig.* 62, same plate.

Or, as m B is in a vertical plane, it may be measured thus: $k7$, in the vertical vanishing line, is equal ke'; and A r is a vertical line of measures, m is fixed or in the line of measures, therefore draw 7 B produced to r; and mr is equal bd or de, in *fig.* 62, as before.

By similar methods may all the parts of the near and remote sides of the roof be measured. But for the off side, or that which inclines to the picture, the points l, in the vertical vanishing line, and b, in the horizontal vanishing line, must be joined, and the centre and distance determined as in $bo'k$.

Hipped roofs of houses, circular objects, and shadows, are designedly omitted in this little work: the foregoing examples are some of the most useful in perspective. However, we will speak of a useful operation in perspective, which is, having a line given in perspective to draw another line in perspective, making any proposed angle with the former line.

RULE.—Find the vanishing point of the given line, and join this point to the eye, call this the

the parallel of the original of the given perspective line; then, with the chord of 60^0, and the eye as centre, describe an arc, through the parallel of the given line.

Now, according as the required line is to make an angle on the right or on the left of the given line, lay off the degrees of the proposed angle from the parallel of the given line, and mark this limit on the arc so described.

Through this limit, from the eye, draw a line produced to cut the vanishing line, which determines the vanishing point for the required line. Then, from any point in the given line, draw a line to this second vanishing point, and the two perspective lines will make the perspective of the proposed angle.

Here the original angle will be at the eye, and the second line drawn from the eye will be parallel to the original of the required line.

EXAMPLES.

In some of the foregoing examples we have taken a line or lines, although the component parts of objects, to illustrate our meaning; we shall be obliged to do in like manner, in the course of the subsequent examples.

In *fig.* 41, *Plate* 8, let *a d*, the left side of

the square in perspective, next the ground line pz, be given to draw a line from the point a toward the right, perspectively perpendicular to ad.

The vanishing point of ad is o, the centre of the picture.

According to the rule join oe, and eo is parallel to the original of the given line ad, which is A D in the plan.

Imagine a line drawn from e perpendicular to eo, and this line will be parallel to the original of the required line; but the line drawn from e is parallel to the picture; consequently, it cannot cut the picture to produce a vanishing point for the required line; wherefore, draw a line from a, as ab, in the given line, parallel to the intersecting line pz, or to roh, the horizontal vanishing line, and it is the line required.

If ab be produced toward the left, that portion of it will also be perpendicular to ad.

If the proposed angle had been less than 90^0, the line drawn from e would cut the horizontal vanishing line, and produce a vanishing point for the required line.

Let a line be given in perspective, whose original is in a horizontal plane, and the original of the required line in a vertical plane.

In *fig.* 42, let B*a* be the given line, it is required to draw a vertical line from the point *a*, making an angle of 90° with B*a*.

Here *soe* is a vertical vanishing line, *h* the eye, and *o* the vanishing point of the given line B*a*.

Join *ho*, and a line drawn from *h*, the eye, perpendicular to *ho*, will be parallel to the original of the required line; but as in the last example, this line drawn from *h* would be parallel to *eos*, and, consequently, produce no vanishing point; wherefore, from *a* draw a line parallel to *soe*, and this line will be perspectively perpendicular to B*a*, as required.

Should the proposed angle be less than 90°, a line drawn from *h*, making such a proposed angle with B*a*, would cut the vertical vanishing line, and determine a vanishing point for the required line.

The learner is requested to exercise his knowledge in such applications.

In *fig.* 61, *Plate* 15, let B*a*, in the perspective roof, be a given line, it is required to draw a line from the point B, making a perspective angle of 90° with B*a*.

yox is the horizontal vanishing line, *e'* the place of the eye, and *x* the vanishing point of B*a*.

From *e'*, the eye, draw *e'y* perpendicular to

$e'x$; and y is the vanishing point for the required line; $e'y$ is also parallel to the original of the required line.

From B draw B y, which makes a perspective angle of 90° with B a, the given line at the point B, as required.

Lastly, when the given line is inclined to the picture and to the horizon, to draw a line from one extremity perspectively perpendicular to such a given line; and, consequently, perpendicular to the plane in which the given line is placed.

Let ce, in the perspective roof, be the given line, it is required to draw a line from c perpendicular to ce, and, consequently, to the plane cefB.

The vanishing line of cefB is ym, but ce vanishes in the vertical vanishing line mxn; hence, the vanishing point of the required line will be found in mxn.

We have mentioned the method of transposing the eye for such a vanishing line as mxn, but we will here again repeat the method.

To transpose the Eye without mxn.

e' is the original place of the eye; with the radius xe', and centre x, transpose e', the eye, round, to cut the horizontal line yox produced,

which here falls beyond the limits of the paper, and marks this transposed place of the eye.

Draw a line from *m* to the transposed place of the eye, and this line will be parallel to the original of the given line c*e*.

From the transposed place of the eye draw a line perpendicular to the line drawn from *m*; and this last drawn line produced will cut the vertical vanishing line *m x n* produced downward, and determine the vanishing point for the required line; because the last drawn line is parallel to the original of the required line.

But, as before remarked, it is often more convenient to transpose the eye within the vertical vanishing line thus: with the radius *x e'*, and centre *x*, bring down *e'*, the eye, to *h*, on the horizontal vanishing line; then join *h m*, and from *h* draw a line perpendicular to *h m*, produced to cut *m x n*, the vertical vanishing line produced, which determines the vanishing point for the required line as before.

This method produces the same result as the former method; for the line *h m*, and the line drawn from *h* perpendicular to *h m*, with the vertical *m x n*, form a triangle; and if we imagine this triangle to turn on the vertical *m x n* produced, if turned toward the right, it will bring the eye, &c. to their appropriate places on the right.

The vanishing point being thus found, draw from the vanishing point, through *c*, a line, as *c i*, and *c i* is perpendicular to the given line *c e*, and to the plane *c e f* B, as required. By similar methods may any angles be formed with a given line, or with any number of lines in all kinds of planes.

We shall conclude these examples with a few words in explanation of figures 61 and 63.

In *fig.* 61, *Plate* 15, are three vanishing lines: namely, *y o x*, an horizontal vanishing line; for this *e'* is the original or first place of the eye.

m x n, a vertical vanishing line; for this the eye is transposed beyond the limits of the paper in *y o x* produced.

And *y m*, an inclined vanishing line; but the place of the eye is omitted, this being supplied in *fig.* 63.

Now imagine *m x n*, the vertical vanishing line, to be lightly cut, and the imaginary triangle, or *m h n*, now really formed, with the eye in the vertex of this triangle, and turned back on *m x n*.

And also *y o x*, the horizontal vanishing line, lightly cut the vertex of the above triangle, that is *h*, the place of the eye for *m h n*, will coincide with the original place of the eye, at the back of the picture, when folded back.

Imagine W B V, the intersecting or ground line, cut lightly on the back, and this turned up in front, so as to become perpendicular to the picture.

Then the eye is in its true place to view the original objects, and the horizontal plane, and the plane from the eye that produces the vertical vanishing line *m x n*, will be parallel to their originals.

Fig. 63, *Plate* 16, is explained in precisely the same manner, only the inclined vanishing line, *b o' k*, has *n*, the eye, in its proper place; and if *b o' k* be lightly cut in front, and turned over, the other lines as in the last figure, lightly cut, namely, *b o i*, *k i l* in front, and W A V at the back, the point *n* will coincide with *e*, the original place of the eye; and exhibit the plane parallel to the original inclined plane of the near side of the roof.

Some eminent writers on perspective explain these operations by turning the foregoing planes in front, and the ground plane back, but this method requires an exercise of the imagination, not in general adapted to the capacities of beginners.

In *fig.* 63, the inclined vanishing line *b o' k* has *o o' n* for its vertical, which is perpendicular to the vanishing line *b k*.

Here, as before remarked, we suppose *b o' k*

to be produced by the intersection of a plane passing from the original place of the eye, parallel to the original of the near side of the roof; and although here $o\,o'\,n$ is the vertical of $b\,o'\,k$; yet we suppose $o'\,n$ the distance of $b\,o'\,k$ from the original place of the eye, when turned back to coincide with the eye; still having $o'\,n$ the vertical of $b\,o'\,k$; for every vanishing line must have a vertical line.

In this last mentioned case, the whole figure will assume the form of *fig.* 39, *Plate* 7; see also *fig.* 55, *Plate* 13, and *fig.* 60, *Plate* 15, in all which the picture and the spectator assume a vertical position, and the verticals of each are evident.

In *fig.* 63, *Plate* 16, when $o'n$ is turned back to the eye, $o'n$ is in the inclined plane, which produces $b\,o'\,k$, and is perpendicular to $b\,o'k$, because $b\,o'\,k$ is also in the inclined plane; therefore $o'\,n$, which is now perpendicular to $b\,o'k$, will also be perpendicular to it in whatever degree of inclination to the picture $o'\,n$ may be placed.

Because the distance of any vanishing line from the original place of the eye is determined by a straight line, cutting the centre of that vanishing line, and becoming perpendicular to it.

When the Ridge of the Roof is inclined to the Picture.

Draw GK, *Plate* 15, *fig.* 60, indefinitely; draw GH in an oblique direction toward the right, and equal 15 feet; draw HI parallel to GK.

Make G*a* equal 12 feet; draw *a*M parallel to GH, and indefinitely; draw *a*N indefinitely and perpendicular to GK, draw MY parallel to *a*N.

Take *z*, in *a*M, and draw *zf* parallel to GK, and equal 15 feet; but let the pupil make *z* the half of *a*M.

Draw *z*K parallel to *a*N; make *zo* equal 5 feet.

Draw *oe* parallel and equal *zf*; join *ef*.

Through *o* draw *xoy* parallel to *a*M indefinitely on both sides.

Make *a*B equal 7 feet; make the angle AB*a* equal 34°; make BA equal 8 feet; draw BC perpendicular to AB, and equal 10 feet.

Draw CD parallel and equal AB; join AD.

Find *r*, the half of AB, and draw *r*F perpendicular to AB, and equal 1 foot and half; join FA and FB.

Draw FE parallel and equal AD; join ED and EC, and the original roof is constructed; the ridge FE, parallel to AD and BC, inclining to the picture in an angle of 34°.

To put ABCD, the bottom part of the roof, in perspective; this is in an horizontal plane, and will vanish in *x o y*, the horizontal vanishing line.

To find the vanishing Point for BA, *and its Parallel* CD: *the corner* B *cuts the Picture.*

Produce DC to cut the picture in *w;* produce also DA to cut the picture in *v.*

e' the eye of the spectator.

Draw a line from *e'* parallel to the original line BA, and it cuts the picture in *x;* this is the vanishing point for AB and DC, because in the horizontal vanishing line *x oy.*

From B and *w* draw B*x* and *wx.*

From *e'* draw *ey* parallel to the original lines AD and BC; and the point *y* is their vanishing point.

Draw B*y*, cutting *wx* in *c;* draw *vy*, cutting B*x* in *a;* and B*adc* is the perspective of BADC.

To find the Vertical vanishing Line for the Sides of the Roof.

x is the vanishing point of A B, and its parallel D C; therefore through x draw nxm parallel to a'N, the vertical vanishing line for B F, C E, and for F A and E D.

To find the vanishing Point for B F *and* C E.

Draw from e', the eye, the line $e'm$ parallel to B F, and it cuts the vertical vanishing line in the point m, the vanishing point for B F and C E; draw B m and cm.

Draw a line from e' as $e'n$, and n is the vanishing point for F A, and its parallel E D.

From n draw na', produced to cut B m in f, the vertex or top of one gable end.

The ridge is parallel to the horizon, and y is the vanishing point for A D; therefore draw fy, cutting cm in e, and fe is the perspective length of the ridge.

From n draw nd, produced to cut cm in e; and B $afdec$ is the perspective of its original A B C D F E, as may be seen by comparing the letters.

Join my, and it becomes the vanishing line of the inclined side or plane B F E C; and if ny

were joined, it would be the vanishing line for A D E F, as before exemplified in another part of this work.

Fig. 61. By mistake, the plan of the roof in this figure is of less dimensions than that in *fig.* 60, but this makes no difference in the process of operation.

Draw the intersecting or ground line, the horizontal line, and the distance of the eye, as in *fig.* 59, *Plate* 14.

Make A B equal A B, as in *fig.* 60.

Draw B A, making the angle *v* B A, equal 36°; draw B *c* perpendicular to B A, and the angle *c* B *w* will be equal 54°, the complement of A B *v*, the angle A B C being 90°; for all lines drawn from any point, as B, on one side of a right line, making any number of angles, their sum will always be 180°, the number of degrees in a semicircle.

Make B A equal 7 feet, and B C equal 8 feet; draw C D parallel and equal B A, and join A D.

Produce D A to *v*, and D C to *w*.

Through *e'*, the eye, draw R *e'* P parallel to *y x*, which is called the parallel of the eye, because parallel to *y x*.

Make the angle P *e' x* equal the angle A B *v*, and draw *e' x;* and *x* is the vanishing point for A B, because *e' x* is parallel to A B, its original.

The vanishing Point x found without the Parallel of the Eye.

The angle P*e'o* is a right angle; P*e'x* is an angle of 36°; its complement is *oe'x*: hence we may use the complement of the given angle *v*BA; therefore make the angle *oéx* equal 54°, the complement of the angle which the original line AB makes with the intersecting line *wv;* and the vanishing point *x* is determined as before.

The angle CB*w* is equal 54°; its complement is 36°: therefore make the angle *oéy* equal 36°; and the vanishing point *y* is determined as before.

Any number of parallel lines, not parallel to the picture, can have but one vanishing point.

Now draw B*x* and *wx;* draw B*y*, cutting *wx* in *c;* draw *vy*, cutting B*x* in *a;* and B*cda* is the perspective of BCDA, the plan of the roof.

To find a vanishing Line and vanishing Points for the Roof.

If an original object consist of planes, and the perspective of such an object be required, we must first find vanishing lines for the planes, and then find vanishing points for their intersections.

And as in the original object the planes are united by the lines forming their intersections, so in perspective the vanishing lines of those planes are united by the vanishing points of their intersections; and hence the reason why we draw a vanishing line through a particular vanishing point, such as x.

The inclination of the sides of the roof to the horizon is respectively in an angle of 22^{0}.

Ba is the end on which the gable end is to appear; its vanishing point is x.

Now, as this is to be united to a vertical plane, draw through x the vertical vanishing line mn perpendicular to yx, produced upward and downward.

In this line will be found the vanishing points for the sides of the roof, one above and one below the horizontal line.

Were there sufficient space, yx might be

produced toward the right indefinitely; then with one foot of the compasses in *x*, and radius *xé*, describe an arc, bringing *é* round to cut *yx* produced; then this point of intersection would be the place of the eye for the vertical vanishing line *nm*.

From this transposed place of the eye draw a line, making an angle with *xy* produced, equal 22°, equal the mutual inclination of the sides of the roof to the horizon.

The line forming the angle being produced, would cut the vertical vanishing line in *m*, the vanishing point for B*f*.

But the point *m* may be found thus:

With the radius *xé*, and centre *x*, describe the arc *éh*, and this will produce the same effect as if brought round to *yx*, produced to the right beyond the vertical vanishing line *mxn*.

Now make the angle *xhm* equal 22°; thus with the chord of 60°, and centre *h*, describe the arc *st*, make *st* equal 22°, and draw *ht* produced to cut *nm* in *m*, the vanishing point for B*f* and *ce*.

Make *xn* equal *xm;* draw B*m* and *cm*.

Draw *na*, produced to cut B*m* in *f*, the top of the gable end.

The ridge is parallel to the side B*c;* hence *y* is its vanishing point; draw *fey*, cutting *cm*

in *e;* draw *nd* produced to cut *cm* again in *e;* and B *cdaef* is the perspective representation of the original roof, whose plan is B C D A.

Plate 16, *fig.* 63. This figure shows the application of the foregoing principles in putting many houses in perspective, all having the same dimensions.

Let the intersecting or ground line, the horizontal line, and distance of the eye, be determined.

Make the distance P A equal I B, *fig.* 61; make the angle D A *v* equal 36°, and the angle B A *w* equal 54°.

Make A D equal 8 feet, and A B equal 10 feet, draw B C equal and parallel to A D; join C D; produce C B to *w*, and C D to *v*.

A B C D is the plan of the first house.

It is supposed sufficient to construct only the visible parts of this house.

Draw A *m* perpendicular to *w v*, and equal 6 feet.

e b is parallel to A B and D C; hence *b* is their vanishing point.

e i is parallel to A D and B C; and *i* is their vanishing point.

Draw A *i* and *m i;* draw A *b* and *m b*.

Draw *v b*, cutting A *i* in *g;* and draw *g c* parallel to A *m*, cutting *m i* in *c*.

Draw *w i*, cutting A *b* in *x;* draw *x y* parallel

to A*m*, cutting *mb* in *y;* and the visible parts of the house, without the roof, are represented.

Through *i* draw the vertical vanishing line *kl* perpendicular to *bi.*

With the radius *ie*, and centre *i*, describe the arc *e*T; and T is the transposed place of the eye, from which make the angle *k*T*i* equal *abc, fig.* 62.

Make *il* equal *ik;* and *l* is the vanishing point for B*c.*

Draw *mk* and *yk;* draw *lc*, produced to cut *mk* in B, the top of the gable end.

The ridge is parallel to *my*, and will vanish in *b:* draw B*b*, cutting *yk* in *s;* and the visible parts of the first house are represented.

To represent other contiguous Houses, having the same Dimensions as the first House.

k, *i*, and *l* will be the vanishing points for all the other houses; and all the gable ends will appear within the triangle *im*B*i.*

From C draw C*k*, cutting B*i* in *d;* from *d* draw *dl*, cutting *mi* in *é;* draw *éi* parallel to A*m.*

The ridges of the roofs vanish in *b.*

Draw *db*, meeting B*c*, as in the figure; and thus may other houses be represented.

Fig. 62 is the construction of the gable end.

QUESTIONS AND PROBLEMS FOR EXERCISE.

1. What is the parallel of the eye, and how is it drawn?

2. An original horizontal line cuts the intersecting or ground line in an angle of 35^0, what angle will this original line make with the parallel of the eye if produced to cut it?

3. Suppose a line drawn from the eye produced to cut an horizontal vanishing line in an angle of $53^0\ 30'$, it is required to find in what angle the original line, to which the former is parallel, cuts the intersecting or ground line? we mean acute angles.

4. What kind of a vanishing line should an original plane have, which is perpendicular to the picture and to the horizon?

5. Suppose an original line in the foregoing vertical plane to make an angle of 45^0 with the horizon, declining from the picture, on what vanishing line is the vanishing point for such an original line, and how is it obtained?

6. The gable end of the common roof of a house is placed in a plane perpendicular to the picture, how is the vanishing point of the ridge of the roof found?

7. In the above roof one side declines from

the picture, and makes, with the horizon, an angle of 38° 30′, in what kind of vanishing line does the roof vanish; and is the vanishing point for the above side above or below the horizon?

8. If an inclined plane cut a vertical plane, in what plane is their intersection?

9. Required the construction of *fig.* 55, *Plate* 13, to twice its present dimensions, making the sides of the roof incline to the horizon in an angle of 30°.

10. Required the construction of *fig.* 56, *Plate* 13, to twice its present magnitude, the inclination of the roof, as in the last.

11. Required the construction of the house on the left in *fig.* 56, twice its present size, the inclination of the roof to the horizon 25°.

12. Required the construction of *fig.* 57, *Plate* 14, to twice its present dimensions; and also of *fig.* 59, to twice its present size: the inclination of the roofs to the horizon 25°: and let the pupil make the street in *fig.* 59, three times its present width.

13. Suppose lines drawn from the eye to the extremities of an original line perpendicular to the picture, and these two lines from the eye to cut the indefinite representation, intercepting the perspective of the original line, in

what kind of plane figure are these parts placed?

14. Suppose the eye transposed to the horizontal vanishing line, to what line must an original line be transposed, in order to determine its perspective representation?

15. If the eye be transposed in order to put an original line in perspective, why must we transpose the original line?

16. Explain the analogy or resemblance of *fig.* 39 to *fig.* 40, *Plate* 7.

17. If from the vanishing point of any line in perspective a line be drawn to the eye, to what original line will this last drawn line be parallel?

One foot of the compasses may be placed in the vanishing point of any line as a centre, and, by extending the other foot, to the place of the eye, the eye may be brought down to the left or to the right, in the vanishing line, in which the vanishing point is; but it often happens for lines inclined to the picture, in acute angles, that the eye, brought down to one particular side of the centre of the picture, is more convenient for cutting off from indefinite representations, a portion equal to an original line, or the converse, than if brought down on the other side.

For the more remote the eye is, when transposed, from the centre of the picture, the more oblique the sections will be by lines drawn from the eye so placed, and consequently more liable to err.

18. Suppose the perspective of an original line be given, and also its vanishing point, a line drawn from the eye to this vanishing point makes an angle with the horizontal line equal 25^0, it is required to find the angle the original line made with the ground line or intersecting line.

19. Supposing the above vanishing point to be on the right of the centre of the picture, it is required to find the most correct method of finding the original line, by bringing down the eye.

20. The wall or plane in which the ends of a roof are placed is perpendicular to the picture; the top and bottom of the wall vanish in the centre of two vanishing lines; which are the vanishing lines?

21. How are the centre and distance of a vanishing line determined?

22. The distance which the centre of a vanishing line is from the centre of the picture, together with the distance of the eye from this vanishing line, and the original distance of the eye from the centre of the picture, are three

lines, which form a triangle, required the kind of a triangle; and an explanation of its parts.

23. In *fig.* 59, *Plate* 14, the vanishing line $5x6$ is parallel to the horizontal vanishing line *hor;* by what kind of a plane was $5x6$ produced?

24. In the foregoing figure, if a line parallel to *hor* be drawn through *y*, in the prime vertical, of what kind of a plane will this line be the vanishing line?

25. In *fig.* 59, *exy* is a vertical vanishing line; will the eye, without being transposed from its original place, be common to *hor* or *exy?*

26. What is a line of measures, and how is it produced?

27. If the ridge of a roof be parallel to the picture, to what line is the line of measures for this roof parallel?

28. For what line or lines is the intersection of the picture with the horizon the line of measures?

29. How is a line of measures posited or placed with respect to the vanishing line to which it belongs?

30. What preparation is necessary to determine the original of a line in perspective by bringing down the eye?

31. Suppose I have the indefinite represen-

tation of a line, and the original line transposed to a line of measures, how am I to determine the perspective of this original line?

32. Required a construction to exemplify the last question.

33. The ridge of a roof is inclined to the picture, how many vanishing lines are required to put the whole house in perspective?

34. To what lines are the lines of measures respectively parallel, in reference to the last question?

35. The intersection of two planes is alternately in each plane; the lower edge of a house in perspective vanishes in the centre of the picture; then we say the vanishing point of the intersection of a horizontal and vertical plane vanishes in the common centre of two vanishing lines; explain the reason, and name the vanishing lines.

36. If a line be given in perspective, and it be required to draw another line in perspective to make a proposed perspective angle with the former, it is required to explain the process of operation, and to repeat the rule for it.

37. The vanishing point of a line falls on the left of the centre of the picture in the horizontal vanishing line; and it being required to

draw another perspective line to make the perspective angle of 90^0 with the former, will the required line have its vanishing point on the left or on the right of the centre of the picture; and in what kind of plane are the originals of the foregoing lines?

38. Two lines in perspective make the perspective angle of 35^0 15′, what angle did their originals make; and what angle will two lines drawn from the eye to the vanishing points of the above lines make at the eye?

39. One line in perspective vanishes in the centre of the picture, the original of which is in a horizontal plane, it is required to draw a line perpendicular to the former, and the required line to be in a horizontal plane, which is the vanishing point of the required line?

40. A vertical line is given in perspective, another line makes the perspective angle of 90^0 with the former, how was this effected: and how many vanishing lines connected with the operation: supposing the given vertical line in a plane, the top and bottom edges of which vanished in the horizontal vanishing line, a little to the left of the centre of the picture?

41. The ridge of a roof is inclined to the picture, and has a vertical vanishing line on the

right for each end of the roof; explain how the eye may be transposed for such a vanishing line.

42. The front of a house, and the ends of its roof have three distinct vanishing points in a vertical vanishing line on the left of the centre of the picture; explain the process by which these vanishing points were obtained. There may be also four vanishing lines for the whole house, it is required to name and explain them.

43. Suppose in the above house the ridge inclines 40^0 to the picture, the roof 35^0 45′ to the horizon, it is required to explain the process of drawing a line perpendicular to the near side of the roof.

44. Required an explanation of *fig.* 60, *Plate* 15, and a construction to twice its present dimensions.

45. Required an explanation of *fig.* 61, and a construction to twice its present size.

46. If we have two original planes intersecting each other, can we find the vanishing point of their intersection without determining the vanishing lines of such planes?

47. Explain the reason why the vanishing point of the intersection of two original planes is always determined by the intersection of their vanishing lines.

48. Required the construction of *fig.* 63, *Plate* 16, to twice its present size.

49. Explain the method of putting any number of houses in perspective, as in *fig.* 63.

Required the construction of a house whose front is inclined to the picture in an angle of 43^0 30′, on the left of the centre of the picture, and the roof inclined to the horizon in an angle of 35^0 30′.

The height of the wall, from the ground to the eaves of the roof, is 7 feet, the front 16 feet long, the end 12 feet wide.

The construction to be made to 2-tenths of an inch to a foot, with a general explanation of all the work.

Note.—As the vanishing points of lines frequently extend beyond the limits of the paper or drawing board, the author necessarily recommends, for the purpose of drawing lines to remote points, an instrument called the centrolinead, invented by Mr. Peter Nicholson, made and sold by Mr. Dolland, St. Paul's Churchyard, London.

ON THE PICTURE.

Picture, from the Latin word *pingo*, to paint; because on it, that is, on the picture, are depicted or drawn the representations of objects, as they appear to the eye of a spectator.

It is by some called the table, from the Latin *tabula*, that means a plank or board on which drawings are made.

It is also called the diaphanous plane, the transparent plane, and transparent medium; but all mean the same thing, that is, a plane like a pane of glass.

Application of the Picture to Practice.

If I take a piece of glass 12 inches long by 8 inches wide, a piece of paper or thin board of like dimensions, two pieces of wood 18 or 20 inches long each, in form like common flat rulers, and one end of each joined together, like a jointed two feet rule; a small brush, and some dark colour, and thus prepared, wish to sketch from nature the following objects:

Two trees, in a line parallel to the intended

position of the picture, and distant from each other the breadth of the intended sketch, which suppose 30 feet.

At some distance, and nearly central between the trees, suppose a house similar to the first house in *fig.* 63, *Plate* 16, and similarly opposed to the picture.

To view the trees and the house I stand at about the distance of 30 feet, because at that distance, in a view like the present, the ground first catches the eye; and I make my station central between the trees.

I first draw a line with the dark colour, about one inch distant from the side edge of the glass, and this I call the ground line.

To find the horizontal Line, Breadth of the Picture, and Distance of the Eye.

I now hold the side edge of the glass to my eye, keeping it parallel to the ground, that is, keeping the plane of the glass horizontal, and move it to me or from me until I bring the outside of the trees within about an inch of the ends of the glass, on the inside.

I apply one end of the ruler to my eye, and mark on it the distance of my eye from the remote edge of the glass.

In this example the ground line is supposed to coincide with the bottom, or lower parts of the trees.

To find the Height of the horizontal Line.

Keeping my head steady, I hold the ruler vertically to the remote edge of the glass, and the apparent distance between the lower part of either tree and that part of it cut by the remote edge of the glass, and mark that apparent distance on the ruler, for the height of the eye.

Being thus far prepared, I draw a perpendicular through the centre of the ground line; lay from the ground line, on the perpendicular, the height of the horizontal line, that is, the height of the eye, which I draw through this point; then lay off from the centre, the distance of the eye upward, and all is prepared to finish the sketch.

This much being done, I hold the glass in a vertical position, and move it until all the lines and points on the glass coincide with their former determined places.

I then trace out the trees, and the various lines composing the house; and this sketch will be the perspective of the original objects.

I produce the bottom and top lines of the walls of the house to the horizontal line; then

if all the lines on each side meet in one point respectively, the sketch so far is correct; and if I draw a line from the eye to each vanishing point, and the lines so drawn make a right angle at the eye, this will be another proof of correctness.

If the gable end be on the right, I draw a vertical vanishing line through the vanishing point of that side of the house; I then produce the lines in the roof to cut this vertical vanishing line, and if the distance of the vanishing points of the roof from the horizontal line be equal, the sketch is truly correct.

To make the foregoing Sketch on Paper or Board.

Determine the ground line, the horizontal line, centre of the picture, and distance of the eye, by the methods used for the glass: the other parts may be obtained by the rulers thus:

Take the breadth of the picture on the ruler from the sketch on the glass; at the proper distance hold the ruler, so that the width on the ruler may coincide with the predetermined extent beyond the trees on the glass; at this distance from the eye, all the subsequent operations should be performed with the rulers.

First hold the ruler in a vertical position,

and take the apparent height of the trees, which mark on the ruler.

Next, with the ruler in a horizontal position, take the width of the branches; transfer those dimensions to the paper or the board.

With the ruler in a vertical position, take the extent from the ground line to the near corner of the house; take also the height of this corner wall, transfer these dimensions to their proper places on the board or paper.

Next hold the edge of one ruler to the near corner of the house, the movable ruler on the right, and the edge downward toward the ground.

Make the lower edge of the movable ruler coincide with the lower edge of the wall on the right; mark on the ruler the apparent length of this lower edge of the wall; transfer these dimensions to their proper places as before, carefully observing the angle made by the movable ruler.

Make the edge of one ruler coincide with the near corner of the house, and the movable ruler being upward, make its upper edge coincide with the eaves of the house, and mark its length as before; transfer these also to their proper places, with attention to the angle made by the ruler.

On the board or paper join the extremities

of the bottom and top edges of the wall, and if this line be parallel to the near edge of the wall, it is so far correct.

Next, hold the ruler so as to coincide with the near edge of the wall as before; but the upper edge of the movable ruler to coincide with the near edge of the roof, and mark its apparent length; these also transfer to their proper places, for one half the gable end.

Again, make the edge of one ruler coincide with the remote vertical edge of the wall, and the upper edge of the movable ruler to coincide with the remote edge of the roof, and mark its length; transfer these dimensions to their proper places, and the gable end is completed: proceed in a similar manner with the parts of the house on the left, first finishing the ridge of the roof.

When this sketch is finished, apply to it the sketch on the glass, and if all their parts mutually agree, we may be assured of their accuracy.

This method, accompanied with the glass, is not, I believe, usually adopted; it may, however, be applied to verify particular sketches.

Windows, chimneys, and other minutiæ, may be finished in like manner.

As any knowledge of perspective is of little utility, unless practically applied to architec-

ture and landscape, we would recommend the pupil to study some works on architecture, such as Mr. Peter Nicholson's, particularly the Youth's Instructor in Drawing the Five Orders, where he will see the newest or most fashionable kind of mouldings, &c.

The works of Mr. John Varley are highly esteemed for their excellence in the composition of landscape; they consist of eight or ten numbers: a number on the study of trees, two numbers on perspective, peculiarly adapted for sketching from nature, and a sheet in which, at one view, the student may match and and compound the various tints in a landscape.

FINIS.

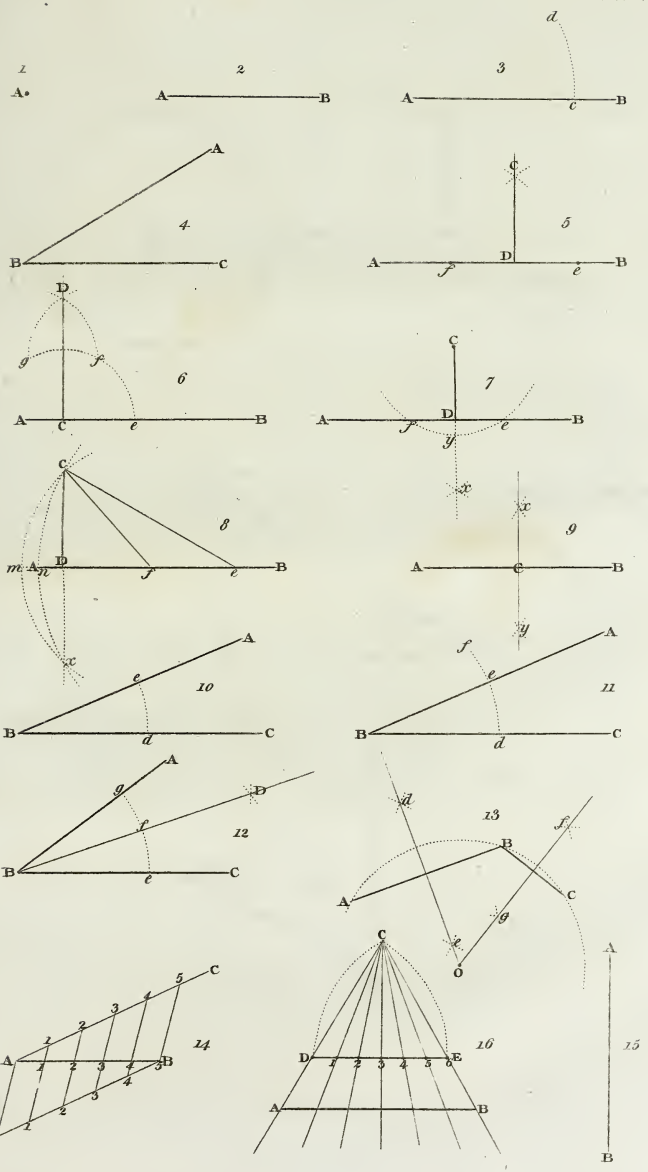
C. and C. Whittingham, College House, Chiswick.

ERRATA.

As in works like the present errors will often unavoidably occur, the reader is requested to correct the following:

Page 9, the seventh line from the bottom, read *Plate* 2.
17, the second line from the bottom, for hx read hn.
34, first line at top, for gc read gC.
36, eleventh line from the bottom, for qm read km.
49, the third line from the bottom, for the point I read the point l.
72, line thirteen from the bottom, for oo' read oP.
72, line eight from the bottom, for Qk read Q K.
92, lines fifteen and sixteen from the top, for $d\,dx$ read $a\,ax$.
93, line eleven from the top, for do read $b'o$.
101, lines thirteen and fourteen from the bottom, for r read T.
110, line fourteen from the top, for ph read pb.

Plate 1.



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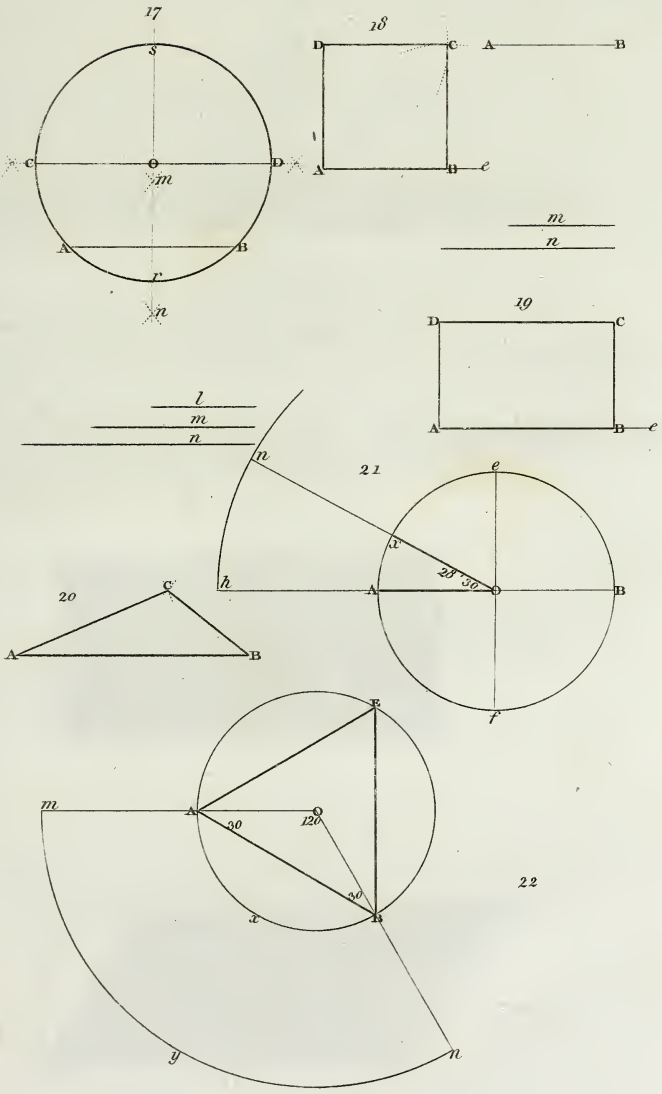

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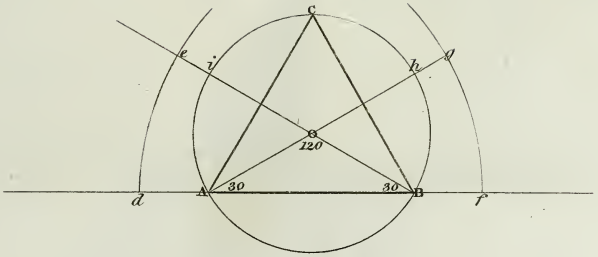

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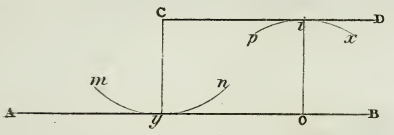

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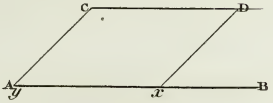

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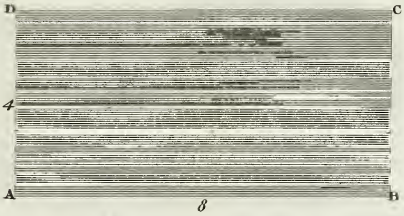

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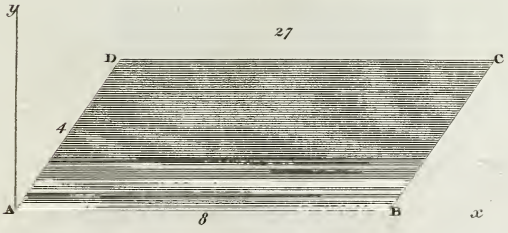

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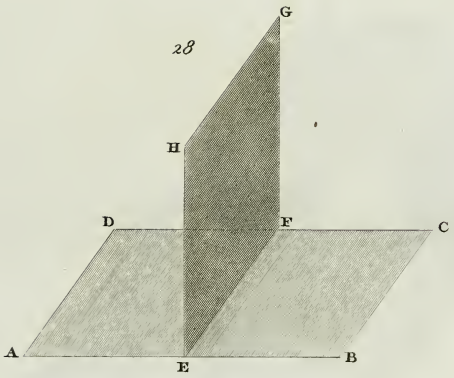

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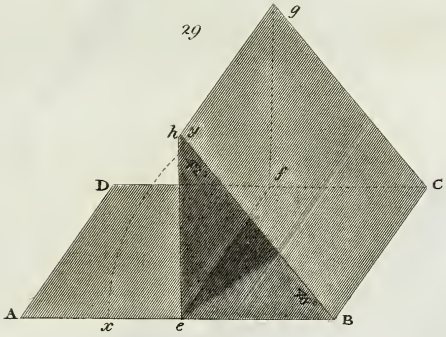

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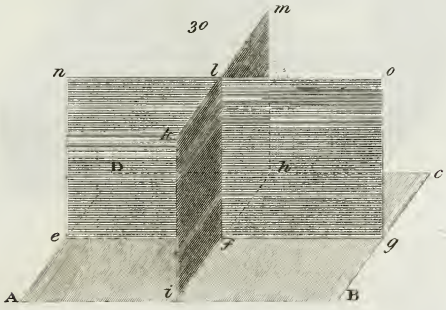

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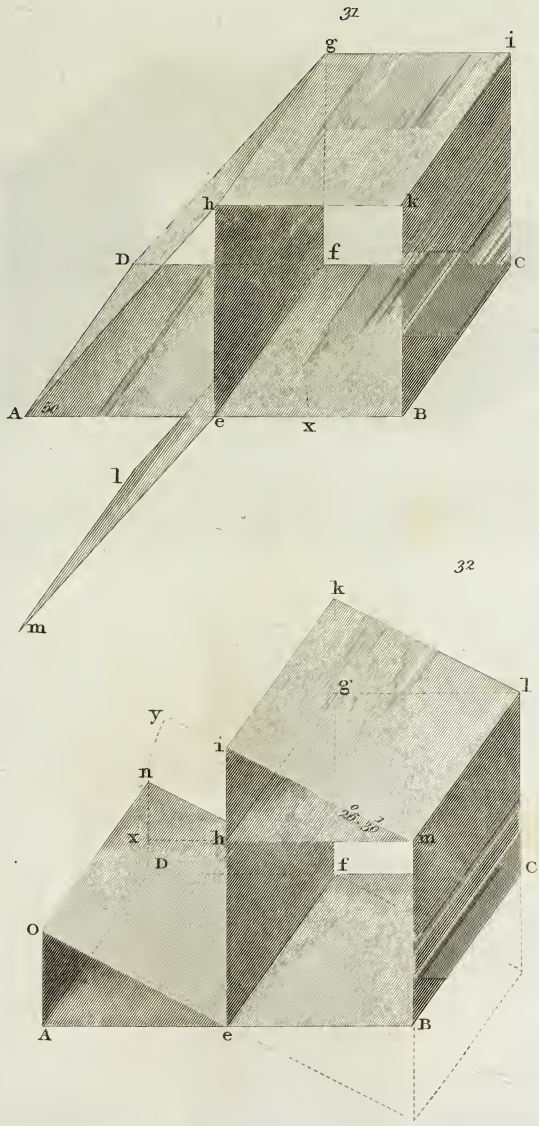

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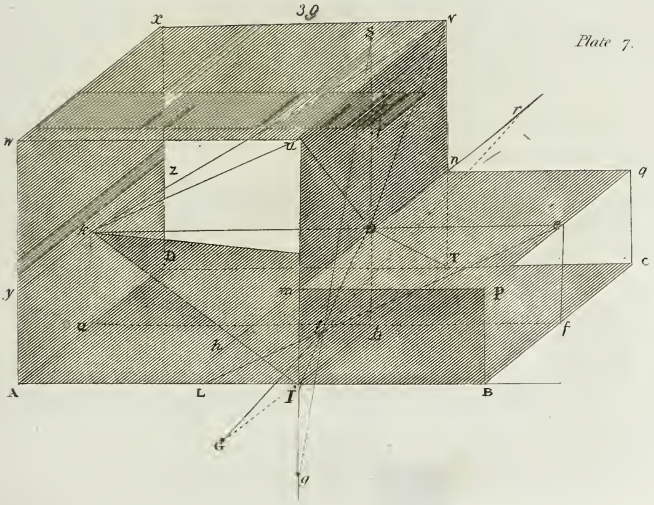
Plate 7.

39.



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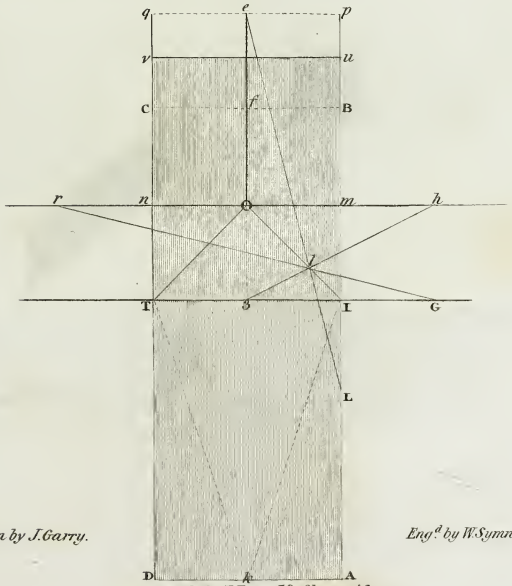

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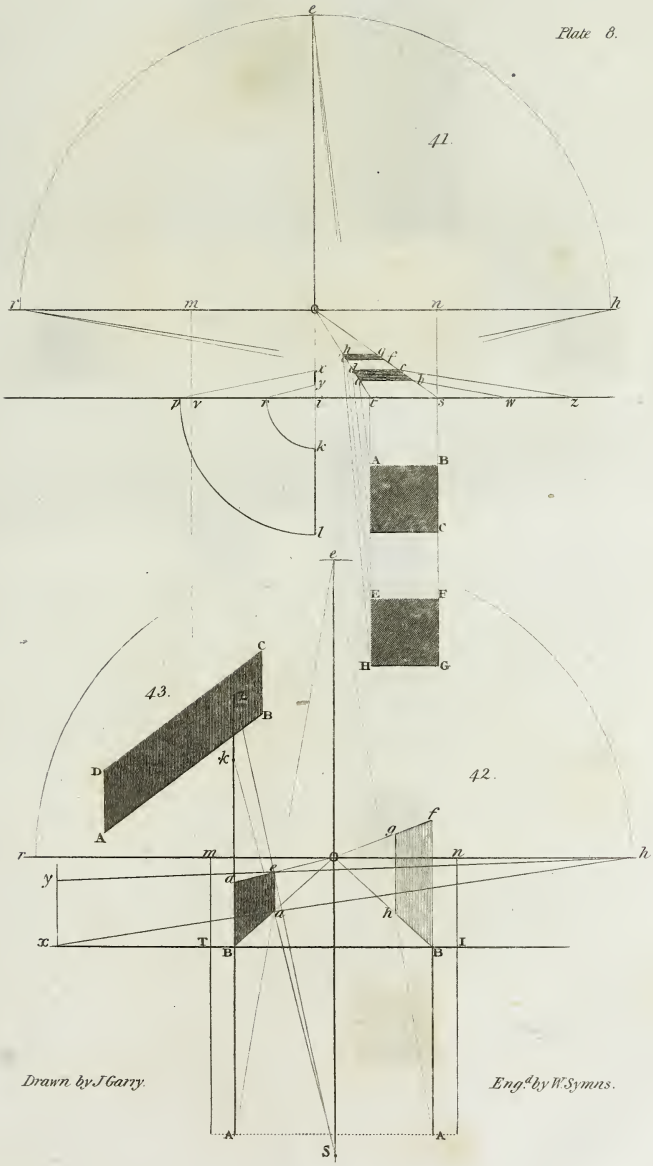
Plate 8.



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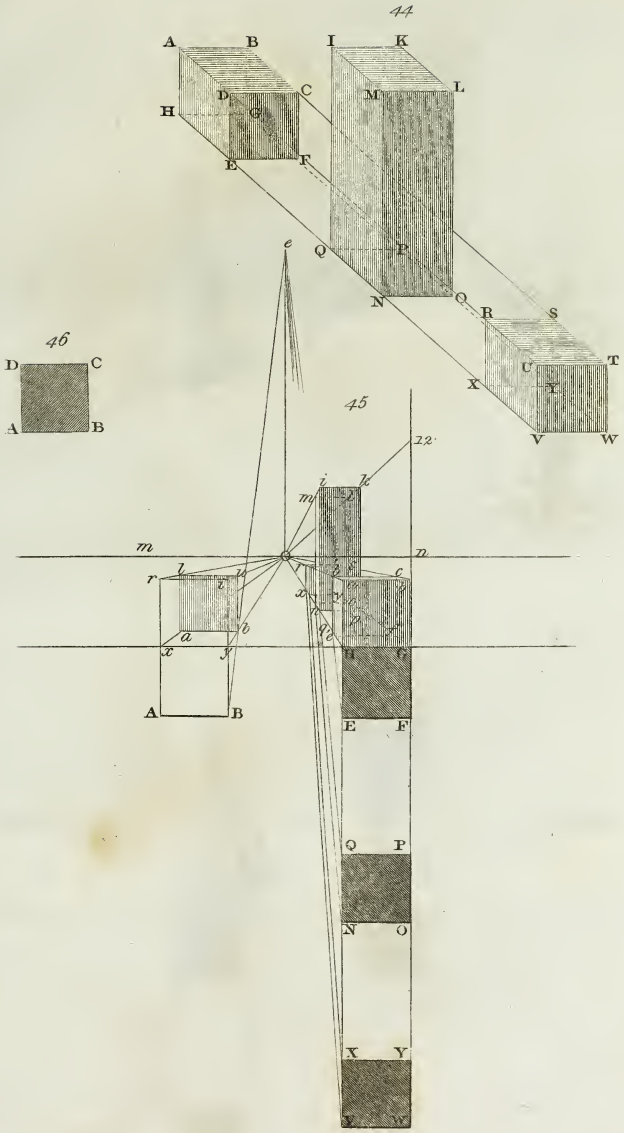

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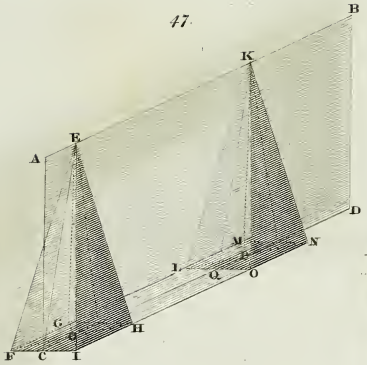

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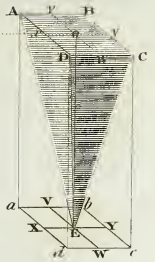

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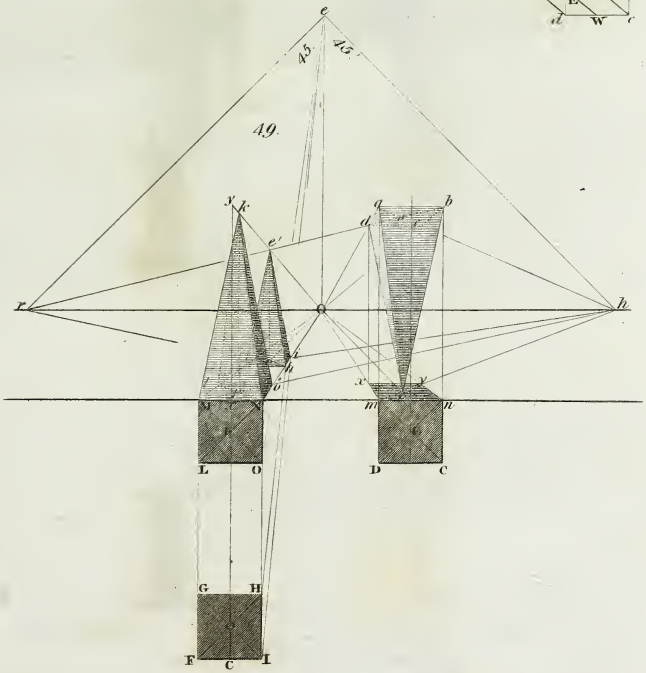

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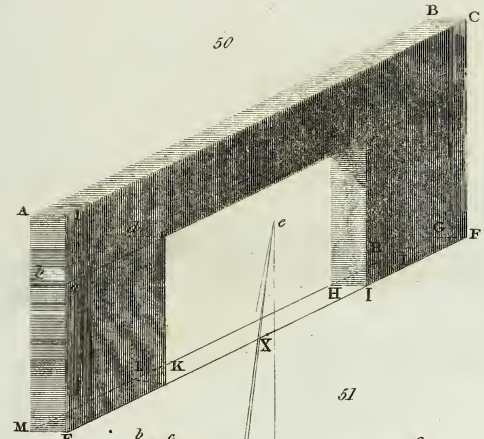

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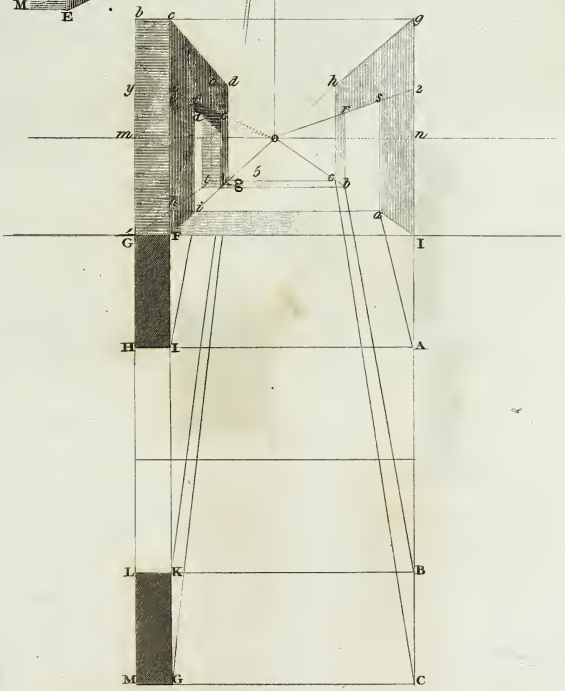

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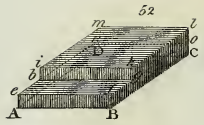

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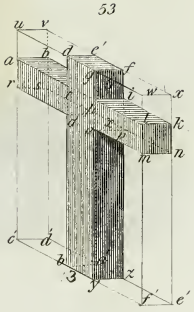

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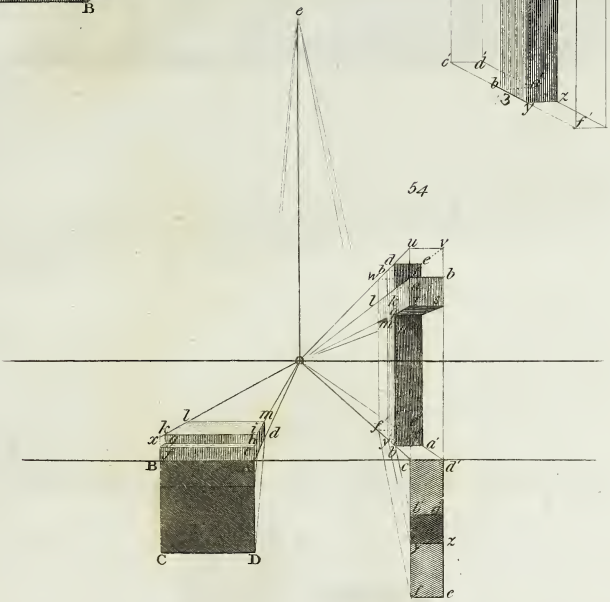

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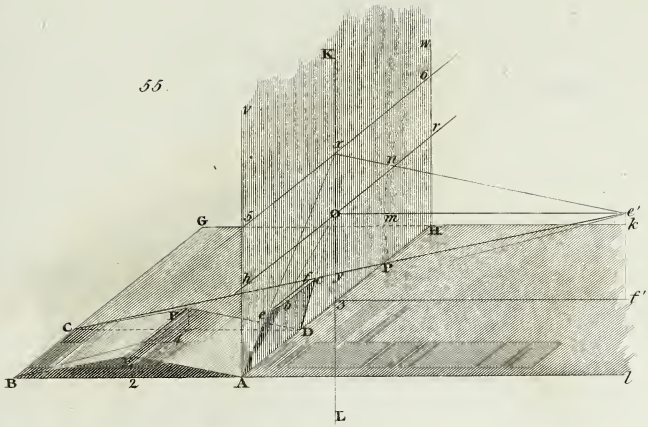

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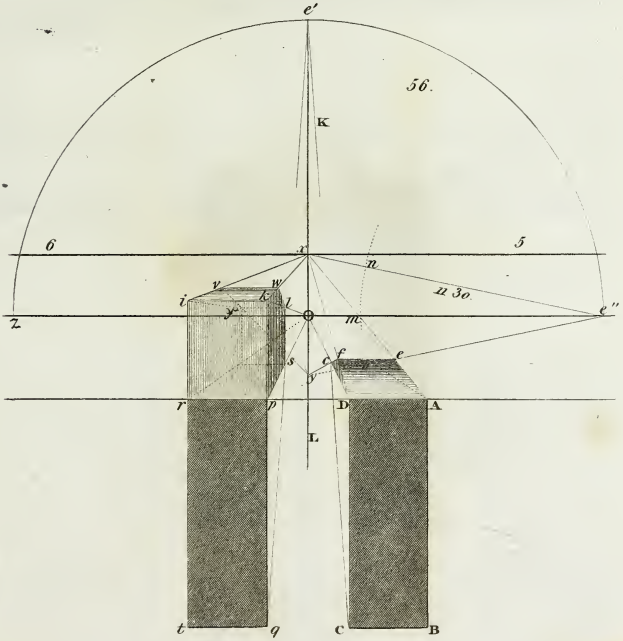

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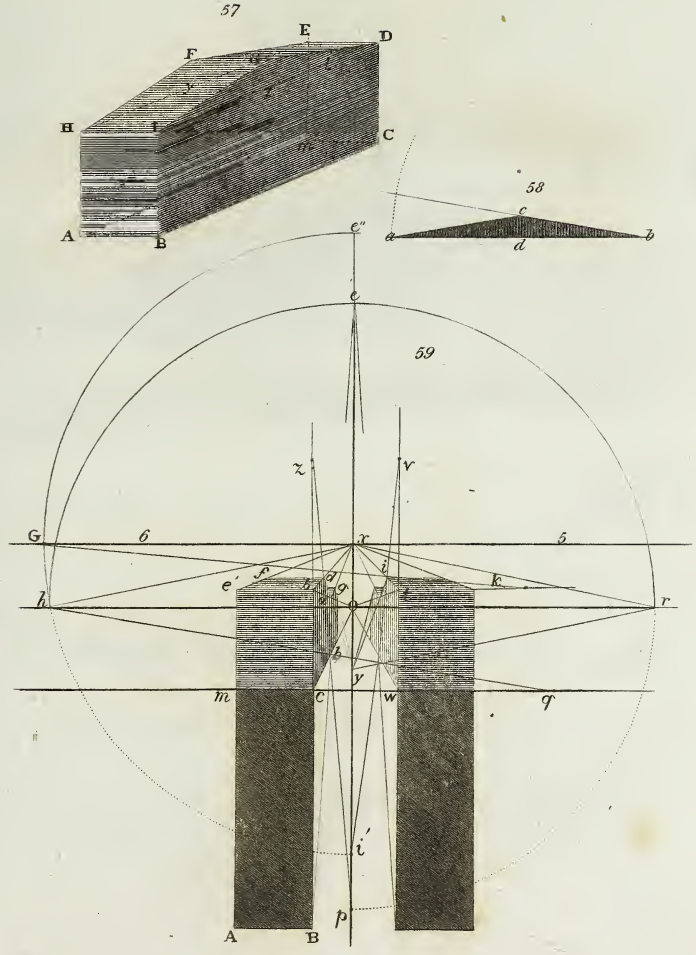

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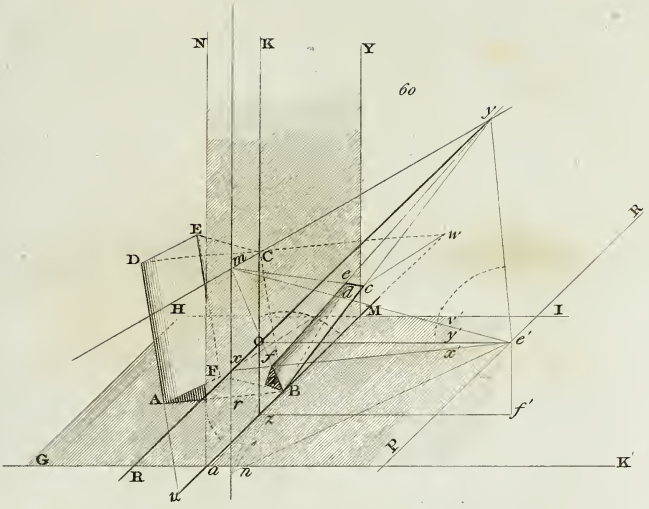

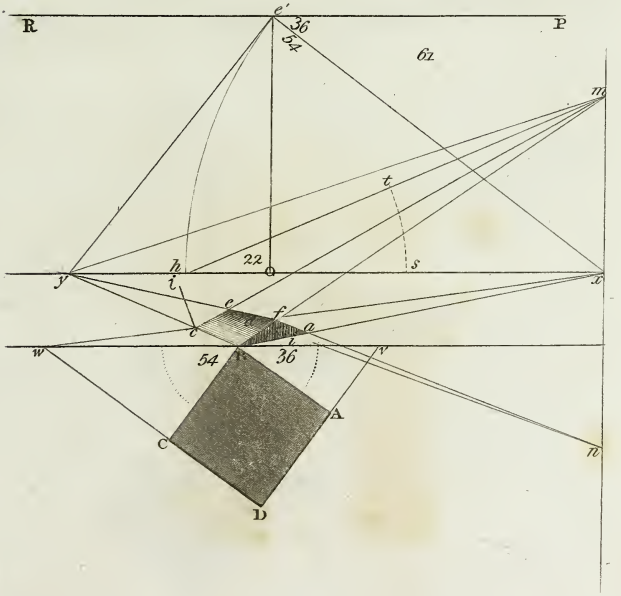

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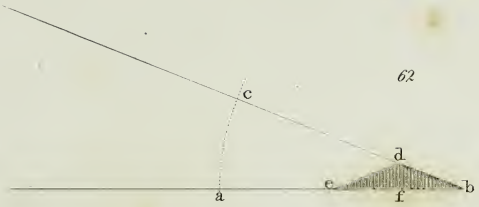

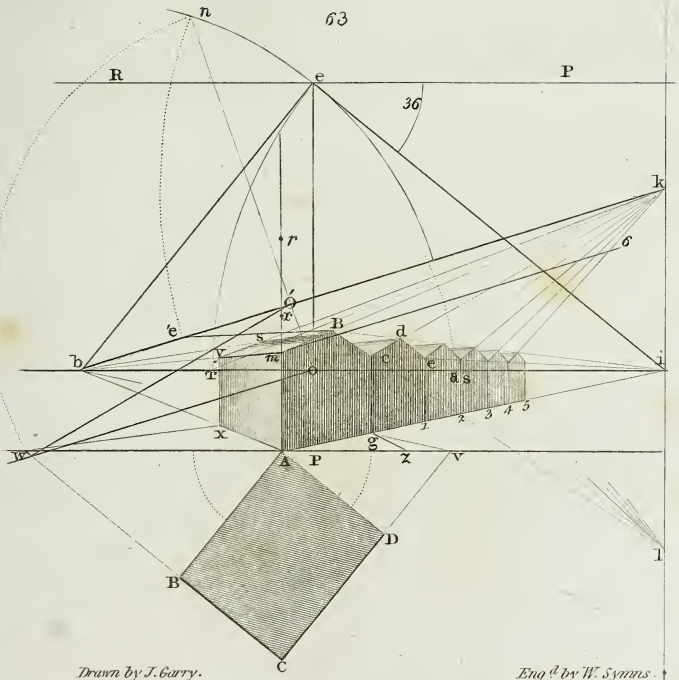

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